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T H E
EXECUTIVE DOCUMENTS

PRINTED BY ORDER OF THE

SENATE OF THE UNITED STATES

FOR THE

FIRST SESSION OF THE FORTY-SIXTH CONGRESS,

1879.

IN THREE VOLUMES.

Volume 1 contains Nos. 1 to 38, inclusive, except Nos. 31 and 37.

Volume 2 contains No. 31.

Volume 3 contains No. 37 and parts.

WASHINGTON:
GOVERNMENT PRINTING OFFICE.

1879.

OBSERVATIONS
OF THE
TRANSIT OF VENUS,

December 8-9, 1874,

MADE AND REDUCED UNDER THE DIRECTION OF THE

COMMISSION CREATED BY CONGRESS.

EDITED BY

SIMON NEWCOMB,
PROFESSOR, U. S. NAVY,
SECRETARY OF THE COMMISSION.

PUBLISHED BY AUTHORITY OF THE HONORABLE SECRETARY OF THE NAVY.

WASHINGTON:
GOVERNMENT PRINTING OFFICE.
1880.

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PART I.

GENERAL DISCUSSION OF RESULTS.

BY

SIMON NEWCOMB,
SECRETARY OF THE COMMISSION.

UNITED STATES NAVAL OBSERVATORY,
Washington, January 17, 1880.

SIR: I have the honor to transmit herewith Part I of the Observations of the Transit of Venus, of December 8-9, 1874, made and reduced under the direction of the Commission created by Congress to superintend the work.

Parts II, III, and IV will follow.

The printing of the work has been ordered by concurrent resolution of Congress.

Very respectfully, your obedient servant,

JOHN RODGERS,
*Rear-Admiral, Superintendent,
President of the Commission.*

Hon. R. W. THOMPSON,
Secretary of the Navy, Washington.

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EDITOR'S PREFACE.

In the issue of the following observations of the Transit of Venus, with their preliminary discussion, the scheme presented by the Astronomer Royal of England to the Royal Astronomical Society, in March, 1875, has been adhered to so far as seemed necessary and practicable. This scheme was worded as follows :

(I). It is desirable that the Observations made in each of the National Expeditions should be printed with the least possible delay, under the superintendence of the head of the Expedition; and should be at once distributed to the principal scientific institutions and libraries, and to the persons officially interested in the Transit.

(II). It is desirable that these Observations should be printed in Quarto, of such size that all could be conveniently bound together. There is little difference in the sizes of the French Memoirs, the Berlin Memoirs, the Smithsonian Memoirs, the Royal Astronomical Society's Memoirs; the size of the *Philosophical Transactions* is somewhat larger.

(III). A brief history of each Expedition should be given, with the names and offices of the persons employed; and description of the localities, accompanied by maps if necessary, and by such statements as may lead to identification of the localities of observation.

(IV). In the accounts of telegraphic operations, it may not be necessary to give details, if the instruments and methods are described, and a few observations are given sufficient for check of the fundamental conclusions; in the accounts of Transits, it is sufficient to give description of instruments and methods, constants of adjustments, and tables of clock-errors, and analogous abstracts of comparisons of chronometers, &c.

(V). But it is necessary to give in the fullest detail everything that bears upon the actual observation of contacts, or upon the observer's impression at the time of making the observation, or upon the micrometer-measures, or upon the photographs and the measures of the photographs, &c.; with sufficient description of the instruments and their adjustments at the time. Clock-time and Local Sidereal Time are to be given for every observation.

(VI). Reference must be made to the place of deposit of the original documents, observations, calculations, and instruments of each National Expedition.

(VII). A large portion of the calculations sketched in the preceding Articles can be best made under the superintendence of the head of each National Expedition, and these should be printed and distributed as part of the Observations mentioned above.

(VIII). It is to be hoped that some astronomer of eminence may undertake to superintend, in some degree, the publications which I have suggested; and to make the final combination of observations and decide on the ultimate result. I can without difficulty fix on the foreign astronomer in whose hands I should be glad to see this work placed.

To these proposals the *Astronomische Gesellschaft*, at its meeting in Leyden, August 16, 1875, added the following :

(IX). Publikationen von Einzelresultaten für die Sonnenparallaxe aus den Beobachtungen des Venusdurchgangs von 1874 sind als die Interessen der Wissenschaft beeinträchtigend thunlich zu vermeiden.

[Publications of separate results for the solar parallax from the observations of the Transit of Venus of 1874 are, so far as practicable, to be avoided as detrimental to the interests of science.]

In reference to IV, it may be said that the time observations are printed more fully than is contemplated in Professor AIRY's plan, partly for the reason that the transit instrument was an essential part of the photo-heliograph used by the American parties.

With reference to VI, it may be stated that the original observations are at present all deposited in a small fire-proof building, erected by the Commission for the purpose, in the grounds of the Naval Observatory. Should the Observatory be removed, as is

contemplated, they will probably be kept in its fire-proof record-room. It may be expected that the reductions will be kept with the observations. The instruments are also stored at the Observatory, with the exception of a certain number temporarily loaned to government offices and to some of the chiefs of parties who took part in the observations. In the general arrangement of the work, the ultimate combination of the observations with those made by other nations has been constantly kept in mind. Therefore, in accordance with the recommendation of the *Astronomische Gesellschaft*, no attempt has been made to deduce an independent value of the solar parallax from these observations. What has been done is to present a comparison of each observation and result with the tables, and to give with each comparison all the necessary data for variations of the various elements involved. The results are presented in such form as to make the effect of any changes in fundamental data as easy as possible. The final step has been to present each result in the form of two sides of an equation, one side of which contains the result of observation, the other the result of theory. The theoretical quantities have been computed from the data of Mr. G. W. HILL, published in Part II of *Papers relating to the Transit of Venus in 1874*. The necessary quantities are, however, given for any change from one set of elements to another.

The numerical reductions have been made, under direction of the Commission, by Mr. D. P. TODD, Mr. W. W. TOWNSEND, Mr. W. F. McK. RITTER, and some other gentlemen only temporarily employed.

It is intended to issue the whole of the observations, with their reduction, in four parts, with the following arrangement of subjects :

Part I. General account of the operations, and reduction and discussion of the observations of the Transit of Venus. This part is issued herewith

Part II. Observations in detail made at each station, with their reduction. This part is intended to follow immediately.

Part III. Discussion of the longitudes of the stations.

Part IV. Measures of the photographs with their reduction and discussion.

NAUTICAL ALMANAC OFFICE,
BUREAU OF NAVIGATION,
NAVY DEPARTMENT,
Washington, March, 1880.

CHAPTER I.

HISTORY OF THE OPERATIONS.

§ 1. ORGANIZATION OF THE COMMISSION.

The first action taken by any public body in the United States, having in view the observation of the Transit of Venus, to occur in December, 1874, is believed to be embodied in a resolution adopted by the National Academy of Sciences, April 16, 1870. This resolution provided for the appointment of a committee to report to the Academy at its next stated session what measures might be necessary to secure the successful observation of the Transit of Venus by American astronomers. The committee appointed in pursuance of this action consisted of—

Professor BENJAMIN PEIRCE, LL. D.,
Rear-Admiral C. H. DAVIS, United States Navy,
And, by invitation—

Commodore B. F. SANDS, United States Navy, the Superintendent of the United States Naval Observatory.

It does not appear that the committee, as thus organized, made any report. At the next session of the Academy the number of the committee was increased; but the necessity for action on the part of the Academy was soon done away with by the action of Congress. A clause in the naval appropriation bill, approved March 3, 1871, appropriated \$2,000 for experiments preliminary to the observations of the Transit, and provided—

That this and all other appropriations by Congress for the observation of the Transit of Venus shall be expended by a Commission, to consist of the Superintendent of the Naval Observatory, the President of the National Academy of Sciences, the Superintendent of the Coast Survey, and two professors of mathematics of the Navy, attached to the Naval Observatory.

The commission, as thus organized, consisted of the following persons:

Rear-Admiral B. F. SANDS, U. S. N., *Superintendent Naval Observatory*, PRESIDENT.

Professor JOSEPH HENRY, *President National Academy of Sciences*.

Professor BENJAMIN PEIRCE, *Superintendent Coast Survey*.

Professor SIMON NEWCOMB, U. S. N., SECRETARY.

Professor WILLIAM HARKNESS, U. S. N.

Changes in the *personnel* have since occurred as follows:

In February, 1874, Rear-Admiral CHARLES H. DAVIS, U. S. N., became Superintendent of the Naval Observatory and President of the Commission in place of Rear-Admiral SANDS, retired.

In May, 1877, Rear-Admiral JOHN RODGERS, U. S. N., took the place of Rear-Admiral DAVIS, deceased the preceding February.

In 1874 Dr. C. P. PATTERSON became Superintendent of the Coast Survey in place of Professor PEIRCE, resigned.

In 1879 President WILLIAM B. ROGERS became President of the National Academy of Sciences in place of Professor HENRY, deceased in May, 1878.

The law of 1871, already referred to, made no provision for anything but some preliminary experiments. During the year following the subject was actively taken up by those interested, and petitions were sent to Congress asking for assistance from the Government. In the legislative branches of the Government the subject was placed in charge of Hon. F. A. SAWYER, of South Carolina, in the Senate, and Hon. JAMES A. GARFIELD, Chairman of the Committee on Appropriations, in the House of Representatives. Through the active interest in the subject taken by these distinguished gentlemen, an item was inserted in the "Sundry Civil Bill," approved June 10, 1872, appropriating the sum of \$50,000 for the purchase of instruments with which to observe the Transit. In the year following a further appropriation of \$100,000 was made for the expenses of the active operations.

§ 2. PLAN OF OBSERVATIONS.

The considerations which guided the Commission in the formation of a plan of observation and the general features of the plan to which they were thus led were as follows:

(1). In studying the subject of contact observations, it was found that all past experience showed this method to be less accurate than had commonly been supposed. It was well known that the latest Transit of Venus (that of 1769) had given a value of the solar parallax decidedly too small, and the most careful research had failed to show satisfactorily any other cause for this error than the difficulty and uncertainty of the contact observations themselves and of the interpretations to be put upon the language of the observers. It was also found that the observations of the transits of Mercury, even those made by the best observers and under the most favorable circumstances, showed important discrepancies. On the other hand, there was reason to suppose that these uncertainties had arisen in part from the want of practice on the part of the observers in noting the phenomena of a transit. No matter how much experience an observer might have in other directions, it was scarcely possible that he could observe more than one or two transits of a planet in the course of his life-time. There was, therefore, reason to hope that, by previous practice on an artificial representation of the phenomenon, the accuracy and certainty of the observation might be considerably improved. It therefore seemed best to choose some method other than that of observations of contacts as the main dependence for the determination of the solar parallax, but at the same time to endeavor to insure, as far as possible, their accurate observation by practicing the observers in connection with an artificial representation of the Transit of Venus.

(2). It was found that the observations of contact might be advantageously supplemented by measures of the distance apart of the sharp cusps of Venus, made with the double-image micrometer, when the planet was nearly on the sun. It was, therefore, determined to equip the principal telescope of each station with such a micrometer—of AIRY'S pattern, as modified by VALZ.

(3). It appeared that any other method than that of contacts must rest upon a determination of the position of the center of Venus relative to that of the sun. Estimates of the sun's semidiameter made by different observers were well known to be so discordant that no reliance whatever could be placed on a determination of the position of the center of Venus relative to the sun's limb. Measures with a filar micrometer appeared to be out of the question, owing not only to the large arc to be measured, but to the personal error in setting a wire tangent to the limb of the sun or of Venus. The differences among observers in noting the time of transit of the two limbs of the sun seemed also to be in the way of obtaining any reliable result from transits across the wires of an instrument. These methods aside, but two ways of determining the position of Venus on the sun were left—heliometer measures and photography.

Notwithstanding the great precision of measures with the heliometer, its application was impracticable, from the fact that the instrument did not exist in this country; and it was not possible to devise, construct, and put into operation a sufficient number of such instruments in the limited time at the disposal of the Commission.

(4). The only resource remaining was to be found in photography; and the Commission were especially predisposed to try this method from the great success with which their distinguished countryman, Mr. L. M. RUTHERFORD, had applied it to astronomical measurements. The question of the form of apparatus to be used next arose, and a correspondence on this subject was opened with Mr. RUTHERFORD, which has been published in Part I of the "Papers" issued by the Commission. At the same time, attention was called by Professor JOSEPH WINLOCK to a method which he had himself invented and put in operation at the Astronomical Observatory of Harvard College. The essential feature of this method is that the photographic telescope is fixed in a horizontal position, while the rays of the sun are reflected into it by means of a heliostat. The telescope may then be made of any required length, and the necessity for a secondary enlargement of the image entirely avoided.*

It was soon seen that this method would offer several striking advantages.

α. The position-angle of the planet on the sun could be determined with much greater precision than by any other method.

β. The linear value of one second of arc on the photographic plate could be determined with great precision by measuring the distance between one "principal point" of the lens and the sensitive plate.

γ. The image to be photographed could be formed in the dark-room itself, in a fixed position above a pier, without any of the inconveniences attending the use of a movable telescope.

A fuller statement of the advantages claimed for the method will be found in the paper referred to, while the details of its application will be given subsequently.

(5). The methods of contact-observations and of photography were combined in an ingenious plan proposed by JANSSEN, whereby the phase of internal contact was itself to be photographed. It was not, however, judged advisable to lay stress on this method, for the reason that the phase, as seen in the photograph, would depend

* Although this method was original with Professor WINLOCK, priority in its application has been claimed by Captain LAUSSEDAI, of France. It does not appear necessary at present to discuss the validity of this claim, which may be well founded without detracting from the merit of the second inventor.

upon the actinic power of the sun's rays and the development of the photograph, as well as upon the actual advance of the phase itself. For instance, a photograph taken at one station thirty seconds after internal contact, with the sun at a low altitude or with a feeble development, might present the same aspect as one taken at the moment of contact at another station with the sun higher, the air clearer, or the photograph more fully developed.

The spectroscopic method of external contact, whereby it had been proposed to observe the approach of Venus on the sun's chromosphere, was believed to be subject to such variations arising from the different degrees of irradiation as not to justify trial.

§3. CHOICE OF STATIONS.

The principal methods of observation being thus reduced to photography when Venus is wholly on the sun, and observations of contacts, it was necessary to choose stations where these methods could be most advantageously applied. The greater the interval of time during which photographs could be taken, the greater the possible number of photographs, and the less the chances of total failure from clouds. It was therefore advantageous to the photographic method to choose stations from which the entire transit would be visible. Such stations would be about equally favorable for the observations of contacts; for although each individual observation of contact would be worth less than if made at the station most suitable for the purpose, this defect would be compensated by the fact that both ingress and egress would be observable.

In order to obtain the best results at the least expense, it was judged necessary that the instruments and methods of observation should, so far as possible, be uniform at all the stations, in order that all the observations might be strictly comparable with each other. It was estimated that the means likely to be placed at the disposal of the Commission would suffice to equip eight stations, and, for the reason just given, it was determined to establish stations only at points from which the whole transit would be observable.

Besides the astronomical considerations which would affect the choice of a station, the chances of fair weather had to be studied. Efforts were, therefore, made to determine in what parts of China and Japan the weather would probably be most favorable in December, and what the meteorological conditions of the region around Kerguelen Island were at the same season. Correspondence was, therefore, opened through the Department of State with the consuls of the United States at Yokohama, Nagasaki, and other points in that part of Asia, with a view of obtaining exact observations of the degree of cloudiness during the months of November and December, 1872 and 1873. The conclusion from these observations was that Nagasaki was the most favorable point in Japan.

In the southern hemisphere it was intended to establish one or two stations on Kerguelen Island or in its neighborhood. Early in 1872, through the courteous intervention of Professor H. A. NEWTON, the Commission was placed in communication with the Messrs. WILLIAMS, HAVEN & Co., of New London, Conn., a firm which occupies a station on Kerguelen Island for hunting the sea-elephant. This island having already been selected as a British station, it seemed preferable to occupy a station on

Heard's Island, in the neighborhood, if the meteorological conditions were not decidedly more unfavorable. An arrangement was therefore made with the firm in question to have a record of the weather at Heard's Island made during the months of November and December, 1872. This record was received in the summer of 1873, and showed that at Whiskey Bay, the sealing station on Heard's Island, the weather was almost constantly so bad as to render observations nearly hopeless. Assurances were also received from the officers of this firm that the weather at their station on Kerguelen Island was decidedly more favorable than at Heard's Island. This station being at a considerable distance from Christmas Harbor (which was supposed to have been selected as the English station), it was determined to occupy it.

It had, at first, been intended to occupy four stations in the northern and four in the southern hemisphere, but an examination of the meteorological conditions of the stations showed the weather to be so much more favorable in the northern than in the southern hemisphere that it was decided to make an unequal division, occupying three northern and five southern stations. The points finally chosen by the Commission were as follows :

(1) *Wladivostok*, Siberia.—This point was selected in consequence of the favorableness of the weather and an intimation from Director STRUVE, of the Pulkowa Observatory, that an application for permission to occupy it might be favorably received by the Russian Government

(2) *Nagasaki*, Japan, was selected as being favorable meteorologically. An additional reason for choosing it was found in the expressed intention of the French expedition to Japan to occupy Yokohama

(3) *Peking*, China, was selected, notwithstanding that a French station was to be established there, because the records of the Russian Physical Observatory at that point showed that an entirely cloudy day had scarcely ever happened at the period of the year in which the transit was to take place. These records, as was subsequently remarked, took no notice of the dust-clouds which so frequently obscure the air in this part of China.

(4) *Crozet Islands*.—This station being favorable astronomically, and not selected by any other nation, it seemed desirable to occupy it, if only the instruments and stores for the party could be safely landed. Ship Harbor, on the eastern, and therefore the leeward side of Possession Island, was selected as the most favorable landing point.

(5) *Molloy Point*, *Kerguelen Island*.—The sealing station of the Messrs. WILLIAMS, HAVEN & Co., in Three Island Harbor, was selected.

(6) *Hobart Town*, Tasmania.—Meteorological observations had been taken there, showing the climate to be quite favorable for observations during December.

(7) *Bluff Harbor*, New Zealand.—This point was selected as being at some distance from those stations which had been chosen by other nations, and as being readily accessible. Communications received through the Department of State indicating that some point in the interior might be more favorable, the party was not confined to the selection of Bluff Harbor

(8) *Whangaroa Bay*, *Chatham Island*, was selected on account of its having a good harbor, though no certain information in regard to its meteorology could be obtained.

The long line over which the southern stations extended would enable the solar parallax to be determined from the difference of position-angle observed from the two ends, though the northern observations should fail entirely. The stations might, in fact, be roughly divided into three groups, the combination of observations made at any two of which would give a valuable result for the solar parallax.

§ 4. INSTRUMENTAL EQUIPMENT.

A uniform plan of observation being adopted at all the stations, it was desirable that the instrumental equipment should also be uniform. Referring to the description given hereafter for fuller details, it will be sufficient to state here that the outfit of instruments supplied to each station included the photographic apparatus complete; a transit instrument, with a clock, two chronometers and a chronograph, and a five-inch equatorial telescope for observing contacts and occultations of stars by the moon. A fundamental part of the plan was to place the transit instrument in the same meridian as the photographic telescope, in order that the central vertical line of the photographic plate-holder could be set very near the meridian and its small deviations be accurately determined.

The transit instruments were of the broken-tube construction, a prism being placed in the center of the tube, by interior reflection from which the pencil of rays is thrown along the axis; and the image is thus formed at the end of the latter. The detailed plans of the instrument were all devised by Professor WM. HARKNESS, U. S. N., and the construction was carried out under his personal direction. This form of instrument has the great advantages of convenience in observing and of rapid and easy manipulation, but is still subject to the disadvantage of a collimation varying with the zenith-distance of the object observed. In the Pulkowa method of using the instrument the inconvenience arising from this source is obviated by a reversal systematically practiced between each pair of stars, a transit of the pole star being observed before and after each reversal. This plan of observation could not be introduced at the American stations owing to the necessity of keeping the instrument in the meridian. The difficulty in question was therefore not so completely avoided as it might have been had the instrument been intended for use out of the meridian.

One of the chronometers furnished to each station was a sidereal one, breaking the galvanic circuit at every second except the sixtieth of each minute, while the other chronometer was regulated to keep mean time. Several of the parties, however, used other chronometers than these two in the course of their observations.

The sidereal clocks were all made by the Howard Clock Company of Boston, after a plan furnished by Professor WM. HARKNESS. They have gravity-escapements, a construction chosen on the ground that, if the escapement and pendulum were well made, the good performance of the clock would be insured without respect to the wheel-work. The latter might therefore be of a very cheap kind. The principal peculiarity of these clocks is the weight of the pendulum, the jar of which holds about forty-five pounds of mercury. In general, the performance of the clocks at the stations was not satisfactory; but it is believed that this imperfection arose from instability of the supports on which they were mounted rather than from defects of construction.

The chronographs were made by ALVAN CLARK & SONS, and were regulated by the Hipp spring. This construction is inconvenient on account of the great weight required to run the instrument, and the noise caused by the spring. It was, however, preferred to the Bond spring-governor, on account of a supposed liability of the latter to get out of order when used in the field. Were the instruments to be reconstructed, it is probable that some form of conical pendulum would be adopted as a regulator.

The photographic objectives were each five inches in clear aperture, and most of them between thirty-eight and thirty-nine feet in focal length. They were corrected for the photographic rays. The heliostat, by which the rays of the sun were thrown into them, turned by clock-work on a single fixed axis, which was so adjusted that the rays of the sun would be thrown in a nearly constant direction during the whole of the transit. This plan was adopted as a compromise between having no clock motion at all in the heliostat, the adjustment being made by an assistant for each photograph, and the expensive apparatus necessary to throw the solar rays in a direction mathematically constant. The latter construction was the more willingly given up from the fear that any motion by a complex system of wheel-work, accompanied by a slow sliding of parts, as in the Foucault construction, might be accompanied by minute jerks. Probably these fears were entirely unfounded, but there was no opportunity of proving them to be so in time to begin the construction.

The mirrors are seven inches in diameter, of unsilvered glass, and slightly thicker on one side than on the other, in order that the reflection from the second surface may be thrown away from the photographic plate. They were left unsilvered in order to prevent any unequal absorption of heat by the two surfaces of the glass. So far as could be detected, no distortion of the unsilvered glass was produced by the direct action of the sun's rays.*

The plate-holder in the focus of the photographic objective consists of a brass frame, about eight inches square, turning on an axis passing vertically through its center, mounted on a hollow iron pier, and having a spirit-level attached to the top of the frame. A vertical cylindrical hole passes through the axis from top to bottom, in the center of which passed a fine silver plumb-line, the bob of which hung in a basin of water below. A square disk of plate-glass, about three-tenths of an inch thick, was set in one side of the brass frame, so that the plumb-line passed very near its surface. The surface nearest the plumb-line was ruled with a system of horizontal and vertical lines one-half an inch apart, by Professor W. A. ROGERS, of the Observatory of Harvard College.

In taking the photograph the ruled plate was on the side of the plate-holder nearest the photographic objective, the sensitized plate being inserted from the other side. Between the ruled surface of the one and the sensitized surface of the other was a space of about 0.ⁱⁿ16, through the middle of which hung the plumb-line. The images of the plumb-line and of the ruled lines were thus impressed on the plate with each photograph.

* In some of the preliminary experiments the mirror was found to become concave under the influence of the solar rays, but this was traced to the heat from the black iron plate on which the heliostat was mounted. On covering the portion of the plate under the mirror with white cloth or paper, the distortion was no longer perceptible.

Each equatoreal was of five inches aperture, was adjustable to any latitude, and was furnished with divided circles, clock-motion, and double-image micrometer. The clock-motion was regulated by a Bond spring-governor, with an auxiliary "fly", in the event of the governor getting out of order. This precaution was suggested by the extreme liability of the governor to break down.

§ 5. ORGANIZATION OF THE PARTIES.

The instrumental equipment and method of observation being nearly the same at all the stations, the force necessary to conduct the scientific operations should also be similar. After due consideration of the conditions to be fulfilled, it was decided that this force at each station should consist of one chief of party, one assistant astronomer (to be second in rank), one chief photographer, and two assistant photographers. In addition, some of the stations were supplied with a mechanician, and two with additional assistant astronomers.

In selecting the chiefs of parties it was decided that two would be furnished by the Naval Observatory, two by the Coast Survey, one by the Army, one by the Navy, and that two should be taken from outside the government service.

Of the assistant astronomers the Army, Navy, and Coast Survey each furnished those of their own parties, while three were taken from outside the government service. The remaining assistant astronomer was supplied by the Army.

In selecting photographers it was deemed necessary that the chiefs should all be professional practitioners of their art; but the greater part of the assistant photographers were young gentlemen of education, recent graduates of different colleges, who had been practiced in chemical and photographic manipulation.

§ 6. PRELIMINARY PRACTICE.

As an essential part of the plan, all the members of the several parties met at Washington, in the spring of 1874, in order to practice all the operations necessary for the successful observation of the Transit, with the same instruments which they were to use at their various stations. The need was now felt of some one skilled in both chemical and physical manipulation, whose duty it should be to put all the apparatus which had been designed by the Commission in complete working order; to complete such details as were still wanting; to try such experiments as might be necessary for this purpose; and to train the several parties in the photographic operations. Desirous of obtaining the co-operation of one of the two citizens of the country most noted for their success in astronomical photography, the Commission, on February 9, 1874, adopted the following resolutions:

- (1) That Dr. HENRY DRAPER, of New York, be invited to take charge, under the direction of the Commission, without pay, of the work of putting into successful execution the various operations necessary for photographing the Transit of Venus by the methods already decided upon by the Commission, and of instructing the parties in those operations.
- (2) That all the material, appliances, and assistance necessary to this end be placed in his hands during the period of his active engagement in this work.
- (3) That he receive his instructions in such way, or through such channel, as the Commission may from time to time determine.
- (4) That his personal expenses while engaged in this work away from his home be reimbursed during the period of such absence, including the cost of railway tickets for the necessary travel.

The need of this action was rendered more pressing from the backward state of the preparations. The Commission had deferred a decision of the question of the length of the photographic telescopes and the construction and application of the heliostat with a view of obtaining light from experimental trials, which, however, had to be finally abandoned. In consequence, there was no time for such deliberate trial of the apparatus as the members of the Commission would have desired to make had a longer interval been at their disposal. Dr DRAPER not only accepted the invitation thus offered, and devoted several weeks to the service of the Commission, at a great sacrifice of his private interests, and under circumstances which rendered his absence from home extremely embarrassing, but also refused to receive any reimbursement of the expenses incurred in performing the duty.*

Valuable service was also rendered by another department of the Government in the examination of the photographers. The number of applications for the position of photographer and assistant photographer being considerably in excess of the number of places to be filled, the Commission preferred a request to the Honorable Secretary of the Treasury that the photographic office of his department should assist in the selection of photographers. The most promising of the applicants were therefore selected and sent to Mr. L. E. WALKER, the chief of the photographic department of the Treasury, for examination. To this gentleman the Commission is indebted for a large part of the labor of selecting the photographers.

§ 7. VOYAGE OF THE SWATARA.

On May 30, 1874, the preparations for the embarkation of the southern parties were completed, and all their instruments and equipments were shipped on the U. S. S. *Gettysburg*, Lieut. D. G. McRITCHIE, U. S. N., commanding, and transported to New York. The parties, with their equipments, were there embarked on the U. S. S. *Swatara*, Capt. RALPH CHANDLER, U. S. N., commanding, the ship fitted out by the Navy Department for distributing the five southern parties among their several stations.

The following is a brief chronological summary of the movements of the *Swatara* in the execution of this duty:

1874.

June 7.—Left anchorage in New York harbor, and put to sea on the following day.

July 11.—Arrived at Bahia.

July 15.—Left Bahia for Capetown.

August 5.—Anchored in Table Bay, Cape of Good Hope. Communicated with the local authorities and with the English parties transported by Her Majesty's Corvette *Encounter*. It was learned that the destination of the English Kerguelen party had been changed from Christmas Harbor to Three Island Harbor, the fact that the

* Desirous of expressing to Dr. DRAPER their appreciation of his disinterested services, the Commission presented him, with its vote of thanks, a gold medal which it had struck for the purpose. The face of the medal bore the inscriptions: VENERIS IN SOLE SPECTANDÆ CURATORES R. P. F. S. HENRICO DRAPER, M. D., DEC. VIII, MDCCCLXXIV. DECORI DECUS ADDIT AVITO. The obverse contained a relief of the photographic heliostat, surrounded by the inscription: FAMAM EXTENDERE FACTIS—HOC VIRTUTIS OPUS.

latter position had been selected by the American Commission not having been communicated to their Government.

August 17.—Left Capetown for the Crozet Islands.

August 30.—Sighted Hog Island and the Twelve Apostles, the westernmost islands of the group.

On the afternoon of the same day a heavy southwest gale came on and the ship had to be hove to. Next morning the gale abated somewhat and Possession Island was sighted to the southward. At 6.30 anchored on the east side, intending to go to Ship Bay at daylight.

At 4 a. m. on September 1 a gale came on from the northward, obliging the ship to stand off shore. She stood on and off the land between Possession Island and East Island during the entire day. In the afternoon an opportunity offered of examining Ship Bay, during a lull in the wind. It was found to be a dangerous place for so large a vessel, as there was no swinging room inside the bay, and the headlands were only two cables' lengths apart. The nearest anchorage that could be found within a mile from the headlands, was in 20 fathoms of water, and exposed to wind and sea. Entertaining the hope that another day might develop better anchorage, Captain CHANDLER stood off shore some ten miles, and hove to under canvas and banked fires. A heavy gale sprung up during the night, and next morning reckoning placed the ship 37 miles from Possession Island, the wind still blowing heavy from the northeast and northward. The outlook was now so unfavorable that Captain CHANDLER felt obliged to give up the attempt to land the party on Possession Island, owing to the danger of delays in landing other parties, a contingency which had been provided for in his instructions.

September 7.—Anchored in Three Island Harbor, Kerguelen Island. A site for the establishment of an observing station was selected on the north side of Royal Sound, about fifteen miles from its mouth. The ship moved over there, and the landing of stores and materials for Commander RYAN's party commenced on the afternoon of the 10th, at a point called Fresh Water Bay, near Molloy Point. Heavy gales were encountered during this time, in one of which the steam-launch was lost. On September 12 leave was taken of the officers of the party, Commander RYAN, Lieutenant-Commander TRAIN, and Passed Assistant Surgeon KIDDER, and the ship sailed for Hobart Town.

October 1.—Reached Hobart Town, Tasmania. The hospitalities of the city were extended to the ship by the Colonial Secretary in the absence of the Governor. The parties of Professor HARKNESS and Captain RAYMOND were landed here, the latter being the one originally destined for Possession Island. Captain RAYMOND selected Campbelltown, 80 miles north of Hobart Town.

October 10.—Left Hobart Town for Bluff Harbor.

October 16.—Arrived at Bluff Harbor. There Lieutenant BASS, Corps of Engineers, U. S. A., assistant astronomer to Dr. PETERS, came on board, having made the journey by way of San Francisco and spent two weeks traveling in the island. He recommended Queenstown, at the north extremity of Lake Wakatipu, as the most eligible site for the observation of the Transit. By direction of the Colonial Governor,

transportation for the party and its outfit was supplied over the government lines of railway free of charge.

October 17.—Left Bluff Harbor for the Chatham Islands. Stopped at Port Chalmers, hoping to find a pilot familiar with the harbors of Chatham Island, but failed.

October 19.—Reached the western end of Petre Bay, and sailed for the small town of Waitangi, at the head of the bay; afterward made the entrance of Whangaroa Bay and found a safe and smooth anchorage just above Point Borgen. Next day Mr. EDWARD SMITH, of the Coast Survey, chief of the Chatham party, went on shore with Captain CHANDLER and selected a station on the rising ground to the westward of the bay. Observations for comparison of chronometers were made by Mr. SMITH.

October 26.—Left Whangaroa Bay for Port Chalmers, New Zealand.

October 29.—Reached Port Chalmers and communicated by telegraph with Dr. PETERS.

November 1.—Telegraphic comparison of chronometers was made by the parties with Dr. PETERS.

November 4.—Left Port Chalmers and next day stopped at Bluff Harbor to give Dr. PETERS another comparison, but he considered it unnecessary.

November 7.—Left Bluff Harbor for Hobart Town.

November 13.—Arrived at Hobart Town, and remained there until after the Transit.

December 18.—Sailed for Auckland Island, to communicate with the German party.

December 23.—Reached Auckland Island, found the German party well, and made a comparison of chronometers.

December 25.—Sailed for Port Chalmers.

December 27.—Arrived at Port Chalmers, and exchanged signals with Dr. PETERS at Queenstown.

December 30.—Sailed for Whangaroa, Chatham Island.

1875.

January 4.—Reached Whangaroa and communicated with Mr. EDWIN SMITH, chief of party. Took the party on board, and afterward sailed for Port Chalmers.

January 15.—Sailed from Port Chalmers for Bluff Harbor, arriving next day, when Dr. PETERS with his instruments and a portion of his party were taken on board.

January 20.—Sailed for Hobart Town, but put into Port William, Stewart's Island, over night.

January 29.—Arrived at Hobart Town.

February 17.—Sailed for Melbourne. At this point the operations of the ship in connection with the Transit of Venus, after having been conducted by Captain CHANDLER with zeal, ability, and success for a period of eight months, were substantially terminated.

It is proper to add that the reports of Captain CHANDLER speak in the warmest terms of the hospitalities and attentions everywhere tendered, both by the British and Colonial authorities.

The parties designed for the northern stations left two months later, and were transported from San Francisco to Nagasaki by the Pacific Mail steamships. Thence, Professor HALL's party was taken to Wladiwostok by the U. S. S. *Kearsarge*, Commander D. B. HARMONY, U. S. N., commanding. The ship reached her destination on September 7, and the party with their instruments and baggage were landed on September 9.

Professor WATSON's party was transported from Nagasaki to Tientsin by the U. S. S. *Ashuelot*, Commander E. O. MATTHEWS, U. S. N., commanding, arriving September 9. The journey from Tientsin to Peking was made by the regular commercial conveyances.

The detailed reports of the movements and operations of each party will be given in Part II.

CHAPTER II.

PARTICULARS RELATING TO EACH STATION.

§ 1. POSITIONS OF STATIONS.

[The longitudes here given are those provisionally adopted in the following reductions. The discussion of the corrections which they may require will be given in Part III.]

Station.	Astronomical Latitude.	Geocentric Latitude.	Log ρ	Provisional Longitude West from—	
				Greenwich.	Washington.
	o ' "	o ' "		h m s	h m s
Wladiwostok	+43 6 35.6	+42 55 6.6	9.999324	— 8 47 30.9	—13 55 43.0
Nagasaki*	+32 43 21.1	+32 32 53.8	9.999578	— 8 39 30.6	—13 47 42.7
Peking	+39 54 15	+39 42 55.7	9.999405	— 7 45 47.9	—12 54 0.0
Molloy Point (Kerguelen Island)	—49 21 22.1	—49 9 59.1	9.999166	— 4 40 18.1	— 9 48 30.2
Hobart Town	—42 53 24.6	—42 41 56.0	9.999330	— 9 49 20.5	—14 57 32.6
Campbelltown	—41 55 42.9	—41 44 16.5	9.999354	— 9 50 0.1	—14 58 12.2
Queenstown	—45 2 7	—44 50 36.4	9.999276	—11 14 40.4	—16 22 52.5
Whangaroa : (Chatham Island.)	—43 49 3.2	—43 37 33.2	9.999307	—12 13 11.8	—17 21 23.9

* The Nagasaki Telegraph Station, from which longitude-signals were exchanged with Wladiwostok, is 1^s.63 west and 50^{''}.3 north of the Transit-of-Venus Station.

§ 2. NUMBERS AND CONSTANTS OF INSTRUMENTS.

Station.	Number of Transit.	One rev. of its Microm.	Five-inch Equatoreal.	Howard Clock.	Mean Time Chronometer.	Sidereal Chronometer.	Additional Chronometers.
Wladiwostok	1508	68.700	856	629	De Silva 1081	Negus 1519
Nagasaki	1507	69.34	862	622	Penlington 1742	Negus 1503	{ Negus 1563 s. t. Negus 1378 m. t.
Peking	1505	68.605	857	628	Negus 1518
Molloy Point	1497	69.00	859	623	Murray 827	Negus 1539
Hobart Town	1502	68.395	863	625	De Silva 694	Negus 1520
Campbelltown	1503	69.11	860	624	Porter 118	Negus 1536
Queenstown	1504	69.45	858	626	Negus 994	Negus 1470	Bond 335 s. t.
Whangaroa	1506	68.94	861	627	Bond 243	Negus 1527	Bond 387 s. t.
Station.	Heliostat Mirror.	Photographic Objective.	Measuring Rod.	Length of Rod, 62° (Fahr.)	Plate-Holder.	Engineer Level.	
Wladiwostok	VII	I	IV	in. 450.357	7	1490	
Nagasaki	IV	VIII	II	450.437	3	1487	
Peking	III	VI	I	461.425	4	1491	
Molloy Point	VIII	V	III	453.488	1	1494	
Hobart Town	V	II	VII	453.498	5	1510	
Campbelltown	VI	III	VI	451.946	6	1499	
Queenstown	II	IV	VIII	451.491	8	1489	
Whangaroa	I	VII	v	449.485	2	1493	

§ 3. ORGANIZATION OF THE PARTIES.

WLADIWOSTOK.

Professor ASAPH HALL, U. S. N	<i>Chief of Party.</i>
Mr. O. B. WHEELER	<i>Assistant Astronomer.</i>
Mr. D. R. CLARK	} <i>Photographic Assistants.</i>
Mr. T. S. TAPPAN	
Mr. GEORGE J. ROCKWELL	
Mr. F. M. LACEY	

NAGASAKI.

Professor GEORGE DAVIDSON, U. S. C. S	<i>Chief of Party.</i>
Mr. O. H. TITMANN, U. S. C. S	<i>Assistant Astronomer.</i>
Mr. W. S. EDWARDS, U. S. C. S	<i>2nd Assistant Astronomer.</i>
Mr. S. R. SEIBERT	} <i>Photographic Assistants.</i>
Mr. H. E. LODGE	
Dr. FRANK H. WILLIAMS	

PEKING.

Professor J. C. WATSON	<i>Chief of Party.</i>
Professor C. A. YOUNG	<i>Assistant Astronomer.</i>
Mr. W. V. RANGER	} <i>Photographic Assistants.</i>
Mr. E. WATSON	
Mr. B. J. CONRAD	

MOLLOY POINT, KERGUELEN ISLAND.

Commander G. P. RYAN, U. S. N	<i>Chief of Party.</i>
Lieutenant-Commander C. J. TRAIN, U. S. N	<i>Assistant Astronomer.</i>
Mr. D. R. HOLMES	} <i>Photographic Assistants.</i>
Mr. G. W. DRYER	
Mr. IRVIN STANLEY	

HOBART TOWN.

Professor WILLIAM HARKNESS, U. S. N	<i>Chief of Party.</i>
Mr. LEONARD WALDO	<i>Assistant Astronomer.</i>
Mr. JOHN MORAN	} <i>Photographic Assistants.</i>
Mr. W. H. CHURCHILL	
Mr. W. B. DEVEREUX	

CAMPBELLTOWN.

Captain C. W. RAYMOND, Corps of Engineers, U. S. A. *Chief of Party.*
 Lieutenant S. E. TILLMAN, Corps of Engineers, U. S. A. *Assistant Astronomer.*
 Mr. W. R. PYWELL }
 Mr. J. G. CAMPBELL } *Photographic Assistants.*
 Mr. THEODORE RICHEY }

QUEENSTOWN, NEW ZEALAND.

Dr. C. H. F. PETERS *Chief of Party.*
 Lieutenant E. W. BASS, Corps of Engineers, U. S. A. *Assistant Astronomer.*
 Mr. C. L. PHILLIPPI }
 Mr. ISRAEL RUSSELL } *Photographic Assistants.*
 Mr. E. B. PIERSON }
 Mr. L. H. AYMÉ }

WHANGAROA, CHATHAM ISLAND.

Mr. EDWIN SMITH, U. S. C. S. *Chief of Party.*
 Mr. ALBERT H. SCOTT, U. S. C. S. *Assistant Astronomer.*
 Mr. LOUIS SEEBOHM* }
 Mr. OTTO BUEHLER } *Photographic Assistants.*
 Mr. W. H. RAU }
 Mr. SUMNER TAINTER *Instrument Maker.*

* Mr. SEEBOHM died at Bahia, Brazil, during the voyage out and his place was taken by Mr. BUEHLER.

CHAPTER III.

DISCUSSION OF THE PHOTOGRAPHIC OPERATIONS.

§ 1. DESCRIPTION AND USE OF THE PHOTO-HELIOGRAPH.

The following considerations will give an idea of the objects aimed at in the design of the instrument for photographing the Transit of Venus. It is evident that if we could point a very long telescope at any close pair of objects in the heavens, and, at the moment of taking their photographic images in the focus, photograph a meridian line passing through or near either of them, we should at once be able to determine their angle of position by the relation of the meridian to the line passing through their centers. It is also evident that if we could measure the distance from the second principal point of the objective to the sensitized plate, we should, by comparing this distance with that of the images, be able to determine the angular distance of the latter from each other.

In using a telescope pointed at the sun or other heavenly body, neither of these requirements can be fulfilled. But, by using a horizontal telescope fixed in the meridian, and throwing the rays of the sun into it with a heliostat, the first condition—that of photographing a meridian line—can be potentially fulfilled, since a vertical line can be photographed, and the relation of this line to the meridian can be accurately determined; and the last—that of measuring the length of the telescope—can be really fulfilled. Thus, position-angle and distance can be determined with equal ease and certainty.

The essential parts of the photo-heliograph are shown in figures 1 to 5, inclusive, figures 1, 2, 3 being the heliostat and the clock-work by which it is moved; and 4, 5 the photographic apparatus. In explanation of its construction, it may be remarked that, under favorable circumstances, no clock-motion of the heliostat is necessary. Experience shows that a photograph of the sun can be taken in a small fraction of the hundredth of a second, so that the want of definition produced by the diurnal motion will be entirely lost in the necessary effects of irradiation. Without clock-work, it would be necessary for an assistant to adjust the position of the reflector by tangent screws for each photograph, and this would have been very easy. There are, however, two contingent circumstances which might make it imprudent to trust entirely to the motion by hand. One is that, in the event of the photographs having to be taken in moments of sunshine during a cloudy day, the sun might, after a brief interval, be covered again by the clouds before the assistant had time to make an adjust-

ment. The other is that, owing to mist or low altitude, a much longer interval than the normal time might be required for the photograph. As the result afterward showed, the photographs at Kerguelen and Peking would probably have been entirely lost from this cause had no clock-motion been applied to the heliostat.

On the other hand, the construction of the apparatus necessary to give the mirror the movement which would throw the reflected ray in an invariable direction was, during the limited time at the disposal of the Commission, entirely impracticable. A middle course was therefore adopted, and a construction of the apparatus decided upon which would be inexpensive, and at the same time throw the ray in a direction so nearly constant that the necessary adjustment would involve but little trouble. The general principle on which the apparatus should be constructed was prescribed by the Commission, and the details, including the entire clock-movement proper, were worked out by Messrs. ALVAN CLARK & SON, the makers. These details are shown in figures 1, 2, and 3. A hollow iron pier, *A*, Fig. 1, about 13 inches in diameter and 10 feet long, is set firmly into the ground (and embedded in masonry when practicable.) Surmounting this pier is a flange, *B*, on which is firmly screwed a bed-plate, *C*, 1 inch in thickness and 33 inches in length, from north to south. Its greatest width is about 16 inches, and it tapers towards each end, so as to have the shape of a coffin.

Upon the northerly end of this coffin-shaped bed-plate rests a triangular-shaped base, *D*, which is supported upon three leveling-screws, by which it may be adjusted to the level. Firmly affixed to it is a massive standard, *E*, through the upper end of which passes the movable horizontal axis, *x*, upon which the hollow cylindrical sheath, *F*, can revolve in a vertical plane. The sheath *F* may be clamped in any of its positions by the nut *a* on the end of the axis *x*.

Through the hollow cylinder *F* passes a second axis, *z*, which at one end is grasped by the clamp, *G*, which may be clamped by the thumb-screw, *h*. These details may be more clearly seen in Fig. 3. The lower end of the clamp *G* terminates in an arc of a circle, upon the edge of which is cut a female screw. An arm, *M*, clasps the cylinder *F*, which may be clamped and adjusted by the set screw *h'*. This arm supports the wheel *W*, by which the motion is communicated from the clock-work. On the axle of this wheel is a ball-joint working in a socket within the bearing *s*, on the lower end of arm *M*; the axle terminates in a screw, *b*, which works in the female screw on the edge of clamp *G*, being pressed upwards and retained in gear by the spring *c*. As the screw *b* merely rests against the clamp *G*, when the latter has moved to the end of the thread, it may be thrown out of gear by simply depressing the end of the screw, and the clamp *G* then moved back and reclamped.

The left-hand end of the axis *z* terminates in a pair of jaws, *H H'*, about 9 inches apart in the clear (only one being shown in Fig. 1); between which, hung on a short axis, *y*, is the circular frame *J*, within which is placed the reflector *K*. In Fig. 1, the axis *z* is inclined 30° to the horizon, and the frame *J*, shown edgewise, at the angle of 60° . In Fig. 3 the axis *z* is level, and the frame inclined at an angle of 45° , showing a portion of the reverse side of the reflector. One end of the arm *d* encircles the axis, *y*, to which it may be clamped by the thumb-screw *e*, while the other end contains a ball-and-socket joint, *d'*, through which passes the thumb-screw *f*, which

also passes through another ball-and-socket joint, g , on the jaw H . By this arrangement the inclination of the reflector K to the axis z may be altered.

It is thus seen that the reflector can be turned upon three axes, one of which, however, does not pass centrally through it, viz :

(1) The horizontal east and west axis x , by which the entire movable portion of the heliostat may be revolved in the plane of the meridian, and clamped at any angle.

(2) The axis z in the plane of the meridian, but usually inclined to the horizon, and passing centrally through the reflector. This axis is intended to be adjusted by turning upon axis (1) at such an angle that the motion of the reflected ray shall be as small as possible during the observations.

(3) The axis y in the plane of the reflector, at an arbitrary elevation, and at right angles to the axis z .

As the elevation of the center of the reflector is altered by the revolution of the heliostat on the axis x , it may be readjusted to the proper level by the capstan-head screws supporting the plate D . The range of inclination required for the heliostat is so small that an adjustment may readily be made in this way.

On the southerly end of the bed-plate C is attached by the screw l the standard L ; and to insure firmness, two projections from the standard enter the hollow cylinders C' C'' , cast on the bed-plate C , the space between being filled with cement. To the face of this standard is screwed the cell I , which holds the objective. These details are omitted in Fig. 3 to avoid confusion, merely the outline of the standard L being partly shown in dotted lines. The centers of the reflector and objective should be adjusted to the same level.

The clock-work, by which the heliostat is made to revolve on the axis z , is very simple. It is shown in Fig. 2, and consists of a box, N , of thin sheet-iron screwed together; from the top of which, suspended from a hook, is a conical pendulum, P , which can be adjusted or regulated by the usual screw on the lower end of the pendulum-bar. Under its point of suspension is a vertical shaft, O , pivoted at its lower end and supported above by the column Q ; to its upper part is attached, by the two ends, a bent or doubled wire, o , forming two guides, between which the lower end of the pendulum P may revolve in a horizontal circle. Upon the shaft O are cut the threads of a screw, in which work the teeth of the gear-wheel R . The shaft of this wheel R passes outside of the box N , and bears on it a belt-wheel, W' , from which motion is imparted to the wheel W by a suitable cord or belt. On the same shaft is also a small gear-wheel, r , shown in dotted lines in the figure, working in the gear-wheel r' , on the axis r'' . This axis r'' also bears the ratchet s' , engaging a pawl on the wheel r' and the barrel s'' , on which is wound, by a suitable wrench, the cord by which motion is communicated from the weight T .

The action of these various parts is readily seen by the figure; the weight T imparts motion to the wheel r' , and thence to the wheel W of the heliostat, and also to the gear-wheel R within the box, causing the pendulum P to revolve, the centrifugal force generated throwing it out between the guides o . When near the limit of its motion the end of the rod is caught in the curved arm q and carries it around a vertical axis with a slight friction, thus lessening the motion.

The wire q is centered on the shaft O and can turn on it, but is not a part of the movable works.

The box N is supported upon three leveling-screws, two under the corners of the side shown in the drawing and the third under the middle of the opposite side. A door in the back of the box gives access to the inside.

The box N rests upon a wooden tripod placed in the ground a few feet behind the heliostat.

The photographic part of the heliograph is shown in Figs. 4 and 5. A hollow boiler-iron pillar, A' , similar to that in Fig. 1, but smaller, is planted in the ground, over which is built the photographic house. This pier is also provided with a flange, B' , on which rests a circular plate, D' , 16 inches in diameter, supported on three capstan-head leveling-screws. These screws pass through the flange, and are secured to the plate by three other screws passing through the latter and acting as pivots in the heads of the leveling-screws. The plate being of the same diameter as the flange, there are cast on the under side of the former four small lugs, which enter the interior of the flange when the leveling-screws are removed, and retain the two parts compactly together for convenience in transportation. Upon the plate D' rests another plate, S , cross-shaped, and secured to the former by four small screws, slotted so as to allow a slight east and west movement for adjustment. From the center of this latter plate S projects downward a hollow cylindrical axis or bearing, U , passing through a circular hole in the center of the plate D' and extending somewhat below the flange B' . Its use is to steady the photographic plate-holder V by means of an axis fitting closely within it, and in which the latter may revolve on a vertical axis. The weight of the plate-holder rests on the two long ends and middle of the cross-piece S ; two small standards rise from the extreme ends of the latter, in each of which is a thumb-screw, i i' , passing through a slot in the standard, and screwing into the base of the movable plate-holder, to clamp the latter in position. The slots permit the frame to have a slight motion around a vertical axis, so that the ruled plate can be adjusted normal to the line from the objective

The plate-holder consists of a frame, v , the side towards the heliostat being shown in Fig. 4, in which is seen an opening, k , seven inches square, with a groove around it, in which is set a piece of plate-glass, m , accurately ruled with very fine horizontal and vertical lines one-half of an inch apart. The glass on which the negative of the sun's image is to be taken is placed in another groove on the further side of the frame, and held there by the spring n . A spirit level, p , is screwed upon the top of the plate-holder frame, for adjusting the same to the level by means of the screws under plate D' . The level is not shown in Fig. 5.

The frame of the plate-holder, including the axis on which it turns, has a vertical opening bored centrally through it from top to bottom. Through this opening passes centrally a plumb-line of delicate silver or platinum wire, fastened at the top to a small plug fitted into the opening, and capable of being turned round in the opening by the hand or pointer t . This wire passes between the ruled plate and the sensitized plate, and supports the bob w , immersed in a jar of water in the interior of the pier, accessible through an opening in the pier.

Thus, in every photograph, an image of the vertical wire is formed on the plate. Lest there might be any curvature to the wire, the hand t is reversed from time to time to eliminate any constant error from this source. The mechanical operation of the plate-holder is very satisfactory. When adjusted to verticality, the screws $i i'$ may be withdrawn and the whole plate-holder quickly turned round without any disturbance of the plumb-line.

Regarding the permanent adjustments, it may be mentioned that the screws by which the bed-plate C is attached to the pier A pass through slots in the former, so that the distance between the heliostat and plate-holder may be regulated. The cross-piece S , on which the plate-holder rests, may, as above noted, be moved laterally by its slotted screws, and thus adjusted to the meridian of the heliostat; while both may be raised or lowered on their leveling-screws. In this way both parts may be adjusted in a horizontal meridian line.

In the use of the foregoing apparatus the transit instrument formed an essential part of it. The arrangement of the various parts was as follows: Supposing a station in the northern hemisphere, the transit was the northernmost instrument. It was mounted in a small portable house upon a pier rising to a convenient height.

Next toward the south was the stand containing the clock-work of the heliostat. The space between the clock-work and the transit-house was wide enough to admit of the convenient setting and use of an engineer's level, for the purpose of determining the level of the photo-heliograph in the manner subsequently described.

Next was set the iron pier and plate carrying the heliostat and objective. These were not placed under a house, it being judged sufficient to inclose the apparatus in a water-proof covering whenever threatened by the weather.

A space of nearly 40 feet then intervened between the photographic objective and the photographic house. The iron pier carrying the reticule and sensitive plate was set inside the latter, as near as practicable to its northern end, the pier extending some depth into the ground. A hole was made in the floor to admit of setting it. A space of about 12 inches was left between the pier and the northern end of the house to admit of convenience in manipulation. A round opening about 8 inches in diameter was made through the side of the house, at the proper level, to admit the image reflected through the objective. This was ordinarily closed by a wooden slide having in it a slit, the width of which could be adjusted to suit the intensity of the solar rays. It was movable back and forth by a pair of springs, so arranged that the motion necessary to admit the rays for an instant to the photographic plate could be made in either direction.

As the apparatus was originally constructed, the photographic telescope was furnished with a tin tube in several sections, which could be extended from the photographic house nearly to the objective. In its interior were set several blackened diaphragms, having circular openings 5 or 6 inches in diameter to admit the rays. To protect this tube from the sun, a shed sloping both towards the east and the west was built over it. But it was finally decided that the entire tube was at least unnecessary and possibly prejudicial to the quality of the sun's image rather than beneficial. It was therefore dispensed with, except one or two sections next the photographic house,

which were retained as an obstacle against the admission of stray light when such light would have proved injurious. The shed, however, was retained for the double purpose of protecting the ground immediately under the axis from the sun's rays and of furnishing a support for the measuring-rod.

The use of the apparatus required the accurate measurement of the distances between the photographic plate, or the glass reticule, Fig. 4, and the photographic objective. The design of an apparatus for this purpose, which should be in every respect satisfactory, was not free from difficulty. It should be portable, and of such a character as to admit of being used under unfavorable circumstances and by persons not expert in measuring. The difficulties were increased by the necessity of measuring through the side of the photographic house and along the optical axis of the telescope. The following arrangement was at length decided upon :

A straight rod, about 12 inches shorter than the distances to be measured, was made by screwing together pieces of 1-inch gas-pipe, each about 5 feet in length. These pieces could be unscrewed and packed in a small box and put together at pleasure, so as to form a rod of definite length. The horizontal pieces of the frame-work which covered the space between the heliostat and the side of the photographic house had notches cut in the upper part, in the same straight line in which the measuring-rod was to be laid. The latter, when in position, was horizontal and about 12 inches above the central axis of the photographic objective. The end nearest the photographic plate passed through a hole in the photographic house only large enough to receive it. It was then completely out of the way and could remain permanently when once set. The inner end was about 6 inches outside the face of the reticule, and the outer end about the same distance from the vertical line above the photographic objective. A fine brass plumb-line could then be suspended over each end of the rod, and the distance between these lines and the respective faces of the reticule and the objective were the only quantities remaining to be measured.

These spaces were measured by a species of jaw-micrometer, designed by Professor HARKNESS. One end of this micrometer could be pressed against the face of the glass from which the measurement was to be made, and the other end, which carried the jaw, brought into coincidence with the plumb-line. The jaw itself consisted of two projecting pieces about an inch apart, pierced horizontally, and in a direction at right angles to the line measured, with pin-holes. By looking through this pair of pin-holes, it could be seen when the plumb-line was in a straight line joining them, and between the two projecting pieces. The jaw could be moved through a space of 2 inches by a rack movement, and the distance read off by a vernier.

The distance between the two surfaces therefore consisted of three parts, namely, the length of the rod and the intervals between the plumb-line and the glass surface read off at each end of it. The entire space between the center of divergence of the objective and the sensitive plate is composed of the following parts :

- (1) From the center of divergence of the objective to its inner face.
- (2) From the inner face of the objective to the first face of the glass reticule, a distance measured in three parts, as just described.
- (3) The thickness of the glass reticule.

(4) The space between the second or ruled face of the reticule and the sensitive plate.

The effective focal length made by all these parts is to be slightly diminished on account of the index of refraction of the reticule.

§ 2. ADJUSTMENTS OF THE PHOTO-HELIOGRAPH.

Besides the purely optical adjustments of the photo-heliograph, designed to secure a good image of the sun upon the plate, certain geometrical adjustments and determinations of errors are necessary in order that the relative positions of the two bodies might be determined from the measures of the photographs :

1. In order that the relation of the vertical line on the photographic plate to the meridian shall be accurately known, it is necessary that the line of collimation of the photographic telescope shall be horizontal and in the meridian of the place ; or, at least, that its deviation from this direction be accurately known. The photo-heliograph was therefore placed at the same elevation and in the same meridian as the transit instrument, in order that the middle vertical ruled line in the focus might be used as a meridian mark for the transit instrument. The *Instructions* required the transit instrument to be set on this mark every evening before the commencement of observations for time, so that the azimuth of the photographic telescope would be equal to either the sum or the difference of errors of azimuth and collimation of the transit instrument.

2. The error of level of the photographic telescope could be obtained approximately by pointing the transit instrument upon it and measuring the apparent zenith-distance of the middle horizontal line on the reticule plate. But as the vertical circle of the transit instrument read only to minutes, and the position angle had to be known within a few seconds of arc, this method was not sufficiently accurate. Each photo-heliograph was therefore furnished with an engineer's level as a part of its subsidiary apparatus. When the error of level of the photographic telescope was to be determined, the transit instrument was pointed upon it, and the horizontal wire of the latter was set so as to cover accurately the center of the middle horizontal line of the reticule plate. The level error of the line of collimation of the photographic telescope would then be equal and opposite to that of the transit instrument, the one objective being too high when the other was too low. The engineer's level was then placed between the object glasses of the two instruments, and was pointed upon the horizontal line of the photographic telescope, and the reading of the level recorded. The level was then reversed, set upon the horizontal wire of the transit instrument, and the reading of its level again recorded. It is evident that if the photographic telescope were truly horizontal, the reading of the level should remain unaltered. If the level adjustment is not perfect, the motion of the bubble will indicate twice the error of level. Several settings were generally made for each determination and the mean taken. There was thus no difficulty in determining the error of level within two or three seconds of arc.

If the error of level was greater than half the distance which the engineer's level was calculated to measure, it is evident that this method could not be applied without

some modification. In this case, when the telescope of the engineer's level was set on the middle horizontal line of the reticule-plate, its spirit-level was so adjusted that its bubble should be near the middle of the tube. It was then reversed and its stand adjusted so that its bubble should be brought into nearly the same position as before. The inclination of its line of collimation would then be the same as when it was pointed into the photographic telescope. The observer then returned to the transit instrument and determined how far it was necessary to move its horizontal wire in order that it should cover the image of the wire of the engineer's level. This distance would evidently be twice the error of level of the photographic telescope.

3. The optical axis of the photographic objective should be directed to the center of the ruled plate. This was effected in the usual way by the observer standing in the photographic room and, holding a candle in his hand, noting the positions of the three images reflected by the surfaces of the objective. If these images coincided when the candle was held in front of the center of the ruled plate, the adjustment was perfect.

4. The reticule-plate was adjusted at right angles to the line of collimation in the same way—or, still better, the observer mounted the engineer's level in the farther end of the photographic house, in the line of collimation of the telescope, and then the plate-holder was turned until the reflected image of the center of the objective of the level was seen through the telescope on the same line with the photographic objective.

5. The distance between the nearer surface of the photographic objective and the unruled surface of the reticule-plate was to be accurately measured from time to time. The method of doing this has already been described in connection with the description of the photographic apparatus.

§ 3. METHOD OF INVESTIGATING THE RELATION BETWEEN THE APPARENT POSITIONS OF VENUS AND THE SUN ON THE CELESTIAL SPHERE, AND THE POSITIONS OF THEIR IMAGES ON THE PHOTOGRAPHIC PLATE.

1. Let us first refer the relative positions of the bodies to a system of co-ordinates in which the axis of x passes from the station in the apparent direction of the sun; the axis of y to a point on the same meridian, 90° to the north; and the axis of z to the west, making a right angle with the other two axes. We suppose the direction of the rays to be corrected for refraction by known formulæ, so that the actual direction of the refracted ray, and not the real direction of the sun, is taken. The equations of a ray coming from the center of the sun and striking the center of the mirror will then be

$$y = 0; z = 0.$$

The equations of a ray, coming from a point of which the angular distance from the center of the sun is s , and the angle of position counted from the sun's north point toward the east is p , will be

$$\left. \begin{aligned} y &= x \tan s \cos p \\ z &= -x \tan s \sin p \end{aligned} \right\} \dots \dots \dots (1)$$

In these equations s and p may represent either the co-ordinates of a point on the sun's limb, or the co-ordinates of the center of Venus. In the former case s will not be constant, owing to the effect of refraction.

2. Let us next refer the ray to a system of co-ordinates in which the axis of X shall be perpendicular to the face of the reflecting mirror, the latter being taken as the plane of YZ , the positive side of which is that from which the ray is reflected. As before, we shall take the axis of Y in the same meridian with that of X , and 90° to the north; and hence the axis of Z toward the west. Let us represent by α and δ the right ascension and declination of the center of the sun, as affected by refraction, and by α' and δ' those of the normal to the mirror, or of the new axis of X . Then, if we represent the new axes by accents, the values of the direction cosines will be

$$\left. \begin{aligned} a &= \cos (X X') = \sin \delta \sin \delta' + \cos \delta \cos \delta' \cos (\alpha - \alpha') \\ b &= \cos (Y X') = \cos \delta \sin \delta' - \sin \delta \cos \delta' \cos (\alpha - \alpha') \\ c &= \cos (Z X') = \cos \delta' \sin (\alpha - \alpha') \\ a' &= \cos (X Y') = \sin \delta \cos \delta' - \cos \delta \sin \delta' \cos (\alpha - \alpha') \\ b' &= \cos (Y Y') = \cos \delta \cos \delta' + \sin \delta \sin \delta' \cos (\alpha - \alpha') \\ c' &= \cos (Z Y') = -\sin \delta' \sin (\alpha - \alpha') \\ a'' &= \cos (X Z') = -\cos \delta \sin (\alpha - \alpha') \\ b'' &= \cos (Y Z') = +\sin \delta \sin (\alpha - \alpha') \\ c'' &= \cos (Z Z') = +\cos (\alpha - \alpha') \end{aligned} \right\} \dots (2)$$

If for the moment we distinguish the co-ordinates referred to the new system of axes by accents, the relations for passing from the first system to the second will be

$$\left. \begin{aligned} x' &= a x + b y + c z & x &= a x' + a' y' + a'' z' \\ y' &= a' x + b' y + c' z & y &= b x' + b' y' + b'' z' \\ z' &= a'' x + b'' y + c'' z & z &= c x' + c' y' + c'' z' \end{aligned} \right\} \dots (3)$$

If we now put for brevity,

$$\left. \begin{aligned} \beta &= \tan s \cos p \\ \gamma &= -\tan s \sin p \end{aligned} \right\} \dots \dots \dots (4)$$

the equations (1) for the course of the ray, by substituting the last values of x , y , and z will take the form

$$\begin{aligned} (a \beta - b) x' + (a' \beta - b') y' + (a'' \beta - b'') z' &= 0 \\ (a \gamma - c) x' + (a' \gamma - c') y' + (a'' \gamma - c'') z' &= 0 \end{aligned}$$

3. The ray is now reflected from the surface of YZ , and this reflection being supposed to take place at the origin, it is evident that its effect will be simply to change the algebraic sign of the value of the co-ordinate x' , which corresponds to given values of y' and z' . The equations of the reflected ray may therefore be put into the form

$$\left. \begin{aligned} (a' \beta - c') y' + (a'' \beta - b'') z' - (a \beta - b) x' &= 0 \\ (a' \gamma - c') y' + (a'' \gamma - c'') z' - (a \gamma - c) x' &= 0 \end{aligned} \right\} \dots \dots (5)$$

The equations for the particular ray which emanates from the sun's center are given by putting both β and γ equal to zero, and are, therefore,

$$\left. \begin{aligned} b' y' + b'' z' &= b x' \\ c' y' + c'' z' &= c x' \end{aligned} \right\} \dots \dots \dots (6)$$

4. Now, let us take a third system of co-ordinates in which the course of this ray shall be the axis of X , that of Y being, as before, in 90° greater declination, and that of Z on the equator in 90° less right ascension. If we distinguish the co-ordinates referred to this system by two accents, we shall have for the course of the central reflected ray,

$$\left. \begin{aligned} y'' &= 0 \\ z'' &= 0 \end{aligned} \right\} \dots \dots \dots (7)$$

If we represent by a, b, c , &c., the direction cosines for passing from the second to the third system of co-ordinates, we have

$$\left. \begin{aligned} x' &= a x'' + a' y'' + a'' z'' & x'' &= a x' + b y' + c z' \\ y' &= b x'' + b' y'' + b'' z'' & y'' &= a' x' + b' y' + c' z' \\ z' &= c x'' + c' y'' + c'' z'' & z'' &= a'' x' + b'' y' + c'' z' \end{aligned} \right\} \dots (8)$$

Comparing (6), (7), and the last two equations of (8), we see that every system of values of x', y' , and z' , which fulfills the two conditions

$$\begin{aligned} -b x' + b' y' + b'' z' &= 0 \\ -c x' + c' y' + c'' z' &= 0 \end{aligned}$$

must also make

$$\begin{aligned} a' x' + b' y' + c' z' &= 0 \\ a'' x' + b'' y' + c'' z' &= 0 \end{aligned}$$

and vice versa. Hence each equation of one set must be derivable from the other set

by a linear transformation. If we call h, h', k and k' the coefficients for transforming the first set into the second, we shall have

$$\left. \begin{aligned} a' &= -h b - h' c & a'' &= -k b - k' c \\ b' &= h b' + h' c' & b'' &= k b' + k' c' \\ c' &= h b'' + h' c'' & c'' &= k b'' + k' c'' \end{aligned} \right\} \dots (9)$$

If we take the sum of the squares of the first set of equations (9), we find

$$a'^2 + b'^2 + c'^2 = h^2 (b^2 + b'^2 + b''^2) + h'^2 (c^2 + c'^2 + c''^2) + 2 h h' (b c + b' c' + b'' c'')$$

which, by the known properties of the direction cosines, reduces to

$$h^2 + h'^2 = 1.$$

Taking the second set of equations (9), we find by the same process,

$$k^2 + k'^2 = 1.$$

If we take the sum of the three products formed by multiplying each equation of the first set by the corresponding one of the second, we find

$$a' a'' + b' b'' + c' c'' = h k (b^2 + b'^2 + b''^2) + h' k' (c^2 + c'^2 + c''^2) + (h k' + h' k) (b c + b' c' + b'' c''),$$

which, by the known properties of the direction cosines, reduces to

$$h k + h' k' = 0.$$

We thus have three equations between the four quantities h, h', k, k' , showing that they may all be expressed as a function of a single quantity A , as follows:

$$\left. \begin{aligned} h &= -\cos A & h' &= \sin A \\ k &= -\sin A & k' &= -\cos A \end{aligned} \right\}$$

These values being substituted in (9) give

$$\left. \begin{aligned} a' &= b \cos A - c \sin A & a'' &= +b \sin A + c \cos A \\ b' &= -b' \cos A + c' \sin A & b'' &= -b' \sin A - c' \cos A \\ c' &= -b'' \cos A + c'' \sin A & c'' &= -b'' \sin A - c'' \cos A \end{aligned} \right\} \dots (10)$$

These equations give, at once,

$$\left. \begin{aligned} a'^2 + a''^2 &= b^2 + c^2 \\ b'^2 + b''^2 &= b'^2 + c'^2 \\ c'^2 + c''^2 &= b''^2 + c''^2 \end{aligned} \right\}$$

from which, by subtracting from the equations

$$a^2 + a'^2 + a''^2 = 1 = a^2 + b^2 + c^2, \text{ etc.,}$$

we find

$$\left. \begin{aligned} a^2 &= a^2 ; a = a \\ b^2 &= a'^2 ; b = -a' \\ c^2 &= a''^2 ; c = -a'' \end{aligned} \right\} \dots \dots \dots (11)$$

The reasons for thus choosing the algebraic signs of a, a' , and a'' will appear presently.

5. Now let us return to equations (5). By substituting for x' , y' , and z' , in the first of these equations, their values in (8), we shall have the equations of any reflected ray referred to the reflected course of the central ray as the axis of X. The first of (5) thus becomes

$$\left. \begin{aligned} & \{(-a a + a'b + a''c) \beta + b a - b' b - b''c\} x'' \\ & + \{(-a a' + a'b' + a''c') \beta + b a' - b' b' - b''c'\} y'' \\ & + \{(-a a'' + a'b'' + a''c'') \beta + b a'' - b' b'' - b''c''\} z'' = 0 \end{aligned} \right\} \dots (12)$$

and the second is formed from the first by changing β into γ and b, b', b'' into c, c', c'' .

By (7) when $\beta = 0$ and $\gamma = 0$, we must have $y'' = 0$ and $z'' = 0$ for all values of x'' . Hence the coefficient of x'' must vanish in this case, which requires that we have

$$b a - b' b - b''c = 0 \dots \dots \dots (13)$$

It is evident from the fact that the normal to the mirror, or the axis of X', bisects the respective courses of the direct and reflected central ray, or the axes of x and x'' , that we have—

$$\begin{aligned} \cos (X, X') &= \cos (X', X'') \\ \text{or,} \quad a &= a \end{aligned}$$

which is the first equation (11), and by substituting the remaining equations (11) in (13) we see that the latter can be satisfied only by the algebraic signs adopted in the former. With these signs (13) becomes

$$a b + a'b' + a''b'' = 0$$

which is true by the property of the direction cosines.

If we now substitute in (12) the values of $a, b, c, a', b', c', a'', b'', c''$, given in (10) and (11), we find

$$\begin{aligned} \cos A y'' + \sin A z'' &= \beta x'' \\ - \sin A y'' + \cos A z'' &= \gamma x'' \end{aligned}$$

If we eliminate, first z'' , and then y'' , from those equations, we find the equations of the reflected ray in the usual form to be

$$\begin{aligned} y'' &= (\beta \cos A - \gamma \sin A) x'' \\ z'' &= (\beta \sin A + \gamma \cos A) x'' \end{aligned}$$

If we substitute for β and γ their values in (4) the equations will become

$$\left. \begin{aligned} y'' &= x'' \tan s \cos (A - p) \\ z'' &= x'' \tan s \sin (A - p) \end{aligned} \right\} \dots \dots \dots (14)$$

6. The next step is to determine the value of A, which depends on the relation of the second and third systems of co-ordinates, but which can be more conveniently made to depend on the position of the first and third systems, and, indeed, on the direction of the sun and that in which the reflected rays of the sun are thrown. In the preceding section we have made no supposition respecting the direction in which the axes of Y'' and Z'' are taken, and A depends solely on this direction. We are

then to find its value when we suppose the axis of Y'' to be on the same meridian with that of X'' , and since this value is independent of p , we can most conveniently find it by supposing $p = 0$, and finding the values of y'' and z'' which correspond to that supposition. Let us consider the two spherical triangles $S M P$ and $S' M P'$, the angles of which are on the following points of the celestial sphere:

S , the sun, in the direction, R. A. = α , Dec. = δ .

P , a point north of the sun at the arbitrary distance s , for which $p = 0$.

M , the normal to the mirror.

S' , P' , the reflected directions of the rays from S and P .

By the laws of reflection, the angles $S M$ and $P M$ are equal, respectively, to $M S'$ and $M P'$, and the angle $S M P$ is equal to $S' M P'$; the two triangles are therefore equal in all their parts, and the angle $M S' P'$ is equal to $M S P$.

Consider next the spherical triangle formed by the north pole O , and the points S and S' . The given parts of this triangle are—

$$O S = 90^\circ - \delta \qquad O S' = 90^\circ - \delta'' \qquad S O S' = \alpha'' - \alpha$$

where α'' and δ'' are the R. A. and Dec. of the point toward which the reflected central ray is thrown. The co-ordinates y'' and z'' of any point on the line P' are given by the equations

$$y'' = x'' \tan s \cos O S' P' \\ z'' = x'' \tan s \sin O S' P'$$

which, compared with (14) for the case of $p = 0$, shows that we have

$$A = O S' P'$$

By the properties of the triangles already shown we have

$$O S' P' = O S' S + M S' P' = O S' S + O S S'$$

That is, $O S' P'$ is the sum of the angles which the great circle $S S'$ makes with the meridians at the points $S S'$. This sum is given at once by one of *Napier's Analogies*, from which we have

$$\tan \frac{1}{2} O S' P' = \frac{\cos \frac{1}{2} (O S' - O S)}{\cos \frac{1}{2} (O S' + O S)} \cot \frac{1}{2} S O S'$$

By substituting for the arcs their expression in terms of δ , δ'' , α , and α'' , this expression becomes

$$\tan \frac{1}{2} A = \frac{\cos \frac{1}{2} (\delta - \delta'')}{\sin \frac{1}{2} (\delta + \delta'')} \cot \frac{1}{2} (\alpha'' - \alpha) \quad . \quad . \quad . \quad (15)$$

from which we obtain A .

7. If the axis of the photographic telescope were exactly in the meridian, and if the image of the reflected central ray coincided with this axis, the equations (14) would at once give us the means of finding the position of any required point on the photographic plate. But, as such coincidence should not be supposed, let us take a fourth system of

rectangular co-ordinates, the axis of X being the central axis of the photographic telescope, which we shall suppose to intersect the photographic plate at its center. The plane of X Y will be assumed as vertical, while that of Y Z will be parallel to the photographic plate. We shall suppose the positive direction of the axis of X to be from the objective to the focus; that of Y to be 90° north of that of X, and that of Z to be, as usual, such that an observer, looking from the positive direction of Z on the plane of X Y, would see a rotation of the axis of Y through 90° into the position of the axis of X to take place in the direction of the hands of a watch.

In finding the directions of these axes relative to those of the preceding system, we shall take advantage of the circumstance that both the axis of X and that of X'' are very nearly horizontal and in the meridian. Let us then suppose the axis of X, as just defined, that is, the central axis of the telescope, to deviate from the meridian towards the axis of Z by the small angle of azimuth a , and to deviate from the horizontal line in the direction of Y by the small quantity b . These deviations will make an elevation of the center of the plate correspond to a positive value of b in the Northern Hemisphere and to a negative value in the Southern Hemisphere, while a deviation of this center west of the meridian of the objective will be positive in both hemispheres. Moreover, let δ_0 be the declination of the true horizon on that meridian toward which the reflector throws the sun's rays (south in the Northern Hemisphere, north in the Southern one) so that we have

$$\text{For the Northern Hemisphere } \delta_0 = \text{N. Lat.} - 90^\circ$$

$$\text{For the Southern Hemisphere } \delta_0 = 90^\circ - \text{S. Lat.}$$

Also, let α_0 be the R. A. of the meridian. Now, reasoning as for the Northern Hemisphere, and neglecting quantities of the second order with respect to a and b , the axis of Y will point north of the zenith by the quantity b , while that of Z will point north of the west horizon in the Northern Hemisphere (or north of east in the Southern Hemisphere) by the quantity a . It is easy to see that, if we suppose the quantities a and b so small that we may regard their cosines as unity, the right ascensions and declinations of the points toward which the respective axes of X, Y, Z are directed will be

$$\begin{array}{ll} \text{X } \alpha = \alpha_0 - a \sec \delta_0 & \text{X } \delta = \delta_0 + b \\ \text{Y } \alpha = \alpha_0 & \text{Y } \delta = \delta_0 + 90^\circ + b \\ \text{Z } \alpha = \alpha_0 - 90^\circ - a \cos \delta_0 & \text{Z } \delta = -a \sin \delta_0 \end{array}$$

The same formulæ really hold true for the case of the Southern Hemisphere. The formulæ will indeed give for Y δ a value exceeding 90° , but this will simply throw the point beyond the north pole to the point

$$\left. \begin{array}{l} \text{R. A.} = \alpha_0 + 180^\circ \\ \text{Dec.} = 90^\circ - (\delta_0 + b) \end{array} \right\} \cdot \cdot \cdot \cdot \cdot \quad (16)$$

The direction of the axis of X'' relative to that of X is to be deduced from the position of the sun's center on the photographic plate. If we represent the azimuthal deviation of this center from the centre of the plate in the direction of the axis of Z,

reduced to arc, by a' , and its angular height above the plate [or below it in the Southern Hemisphere] by b' , the azimuth and error of level of the axis of X'' will be

$$\left. \begin{aligned} a'' &= a + a' \\ b'' &= b + b' \end{aligned} \right\} \dots \dots \dots (17)$$

The right ascensions and declinations of the three axes on the celestial sphere will be, from the definitions already given, taking a'' and b'' so small that we may suppose their cosines unity :

$$\left. \begin{aligned} X'' \alpha &= \alpha_0 - a'' \sec \delta_0 & X'' \delta &= \delta_0 + b'' \\ Y'' \alpha &= \alpha_0 - a'' \sec \delta_0 & Y'' \delta &= \delta_0 + 90^\circ + b'' \\ Z'' \alpha &= \alpha_0 - 90^\circ - a'' \sec \delta_0 & Z'' \delta &= 0 \end{aligned} \right\} \dots (18)$$

Each of the direction cosines is now given by an expression of the form

$$\sin \delta \sin \delta'' + \cos \delta \cos \delta'' \cos (\alpha - \alpha'')$$

where, for α , δ , α'' and δ'' are to be substituted the right ascensions and declinations of the points toward which the several axes are directed, as given above.

By substituting the respective values of δ and δ'' just given, and regarding a , b , a'' , and b'' as quantities so small that we may put

$$\sin a = a \qquad \cos a = 1, \text{ \&c.}$$

the values of the direction cosines are found as follows :

$$\begin{aligned} \cos (X X'') &= 1 \\ \cos (X Y'') &= \sin (\delta_0 + b) \sin (\delta_0 + b'' + 90^\circ) \\ &\quad + \cos (\delta_0 + b) \cos (\delta_0 + b'' + 90^\circ) \cos [(a - a'') \sec \delta_0] \end{aligned}$$

Owing to the minuteness of a and a'' we put $\cos [(a - a'') \sec \delta_0] = 1$, which reduces the last expression to

$$\cos (\delta_0 + b - \delta_0 - b'' - 90^\circ) = \cos (b' + 90^\circ) = -\sin b' = -b'$$

In the same way the remaining cosines are found to be as follows :

$$\left. \begin{aligned} \cos (X Z'') &= -a' \\ \cos (Y X'') &= b' \\ \cos (Y Y'') &= 1 \\ \cos (Y Z'') &= a'' \tan \delta_0 \\ \cos (Z X'') &= a' \\ \cos (Z Y'') &= -a'' \tan \delta_0 \\ \cos (Z Z'') &= 1 \end{aligned} \right\} \dots \dots \dots (19)$$

These formulæ are true for both hemispheres. The relations between the co-ordinates, as referred to these two systems, are therefore

$$\left. \begin{aligned} x'' &= x + b' y + a' z \\ y'' &= -b' x + y - a'' \tan \delta_0 z \\ z'' &= -a' x + a'' \tan \delta_0 y + z \end{aligned} \right\} \dots \dots \dots (20)$$

To find the equations of a reflected ray, these values are to be substituted in equations (14), in which we shall put for brevity

$$\left. \begin{aligned} \beta_0 &= \tan s \cos (A - p) \\ \gamma_0 &= \tan s \sin (A - p) \end{aligned} \right\} \dots \dots \dots (21)$$

thus making the equations of the ray

$$\left. \begin{aligned} y' &= \beta_0 x', \text{ or } y' - \beta_0 x' = 0 \\ z' &= \gamma_0 x', \text{ or } z' - \gamma_0 x' = 0 \end{aligned} \right\}$$

Substituting these expressions for y' , z' , and x' in equations (20), and then solving the linear equations in y , z , and x thus formed, we find expressions for z and y in terms of x . In solving these equations we regard a , a' , b , b' , a'' , γ_0 , β_0 , as small quantities of which we may neglect all terms of the second order when multiplied by y or z , and all terms of the third order when multiplied by x . To find the probable magnitude of the errors thus produced, we remark that the quantities γ_0 and β_0 are, at the most, equal to the sun's semi-diameter expressed in arc, or to .0047, while in the photographs taken by the parties the center of the sun was so near the center of the plate that a' , b' , and a'' were generally less than b' , or .0017. The maximum value of β_0^3 or of γ_0^3 would therefore be .000,0001. But the terms multiplied by x and actually neglected do not contain the third power of β_0 or γ_0 as a factor, the largest being of the form

$$\Delta y = a'' \beta_0 y$$

so that we have, in general, for the limit of error of y or z ,

$$\Delta y \text{ or } \Delta z < .000,007 y.$$

The value of y itself will generally be less than 1100'', hence we shall have, in general,

$$\Delta y < 0''.008$$

a quantity considerably smaller than the extreme limit of accuracy which we can hope to obtain in the measurement of a photograph. The equations of the ray, which we thus obtain, referred to the axis of the telescope, are as follows:

$$\left. \begin{aligned} y &= \{ \beta_0 + b' + (\gamma_0 + a') a'' \tan \delta_0 \} x \\ z &= \{ \gamma_0 + a' - (\beta_0 + b') a'' \tan \delta_0 \} x \end{aligned} \right\} \dots \dots \dots (22)$$

8. By what precedes, we have found expressions for the equations of a ray emanating from any assumed point of the sun's disc, and reflected from the mirror, referred to the axis of the photographic telescope, and a vertical plane passing through that axis. But our equations give only the direction of reflection from the mirror, and do not follow the ray through the object-glass. Moreover, the equations strictly apply only to a ray reflected from the center of the mirror, or from the point which lies on the prolongation of the axis of the telescope. But, since all the rays which emanate from the same point are considered as parallel, both before and after reflection from the mirror, which is supposed to be plane, it follows that the general equation of the rays

emanating from an assumed point of the sun's disk will be found by simply adding arbitrary quantities y_0 and z_0 to the equations (21). That is, if we put, for brevity,

$$\begin{aligned}\beta_1 &= \beta_0 + b' + (\gamma_0 + a') a'' \tan \delta_0 \\ \gamma_1 &= \gamma_0 + a' - (\beta_0 + b') a'' \tan \delta_0\end{aligned}$$

the general equations of the rays, after reflection from the mirror and before striking the objective, will be,

$$\begin{aligned}y &= \beta_1 x + y_0 \\ z &= \gamma_1 x + z_0\end{aligned}$$

Now, Gauss, in his *Dioptrische Untersuchungen*, §§ 1-4, has shown that all such rays having common values of β_1 and γ_1 will come to a common focus at a point whose co-ordinates depend on β_1 and γ_1 and on the curvatures and refractive indices of the lenses. A clear idea of the essential features of Gauss's theory may be found in the following way: Let us consider all the parallel rays which, emanating from a common direction, A, strike the objective or other combination of lenses. After passing through the combination there will be one ray, and one only, which will emerge parallel to its original direction, A. Let us call it the central ray, and conceive of it as isolated from all the other rays. We shall then have a single ray corresponding to the direction A, which we may call the central ray A. Then, by the theory of GAUSS, supposing A to vary, all the central rays which strike the combination from various directions converge toward a certain fixed point, and after leaving it they diverge from another fixed point. These two points Gauss calls "Haupt Punkte," and it is on them alone that the magnitude of the focal image depends. A clearer idea of their properties may be afforded if we call the first of these points the center of convergence and the second the center of divergence. In fact, it is evident that if all the rays from a point meet the central ray in another point, or focus, the image of the point will lie on the line of the central ray. The linear distance of the images of two points will then be proportional to the distance of the focal plane from the center of divergence. If the condition that all rays emanating from a distant point converge to the same focus is not rigorously fulfilled, yet, if the central ray occupies a mean position among them, the same thing will hold true, the distance of the mean images being proportional to the distance from the center of divergence to the plane on which they are formed.

9. We have next to consider the positions of Venus relative to the outline of the solar limb, as they appear on the photographic plate. For the center of the sun, β_0 and γ_0 both vanish; the co-ordinates of the point on the photographic plate which corresponds to the center of the sun are therefore found from (22) by putting $\beta_0 = 0$; $\gamma_0 = 0$. Representing the Gaussian co-ordinates $\eta^* z^*$, which correspond to this point by η_0 and z_0 , we find,

$$\left. \begin{aligned}\eta_0 &= (b' + a' a'' \tan \delta_0) f \\ z_0 &= (a' - b' a'' \tan \delta_0) f\end{aligned} \right\} \cdot \cdot \cdot \cdot \cdot \quad (23)$$

f being the focal length, while the general co-ordinates corresponding to the point whose angular distance from the sun's center is s and position-angle p are

$$\left. \begin{aligned}\eta &= (\beta_0 + b' + (\gamma_0 + a') a'' \tan \delta_0) f \\ z &= (\gamma_0 + a' - (\beta_0 + b') a'' \tan \delta_0) f\end{aligned} \right\} \cdot \cdot \cdot \quad (24)$$

If we represent the co-ordinates relative to the sun's center by η_1 and ζ_1 , we have from (23) and (24)

$$\left. \begin{aligned} \eta_1 &= \eta - \eta_0 = (\beta_0 + \gamma_0 a'' \tan \delta_0) f \\ \zeta_1 &= \zeta - \zeta_0 = (\gamma_0 - \beta_0 a'' \tan \delta_0) f \end{aligned} \right\} \dots \dots (25)$$

Taking the sum of the squares of these quantities, substituting for β_0 and γ_0 their values (21), and neglecting the factors of the fourth order, $\gamma^2 a''^2$, we have, for the distance of the general point from the sun's center,

$$\rho = \sqrt{\eta^2 + \zeta^2} = \sqrt{\beta_0^2 + \gamma_0^2} f = f \tan s \} \dots \dots (26)$$

which shows that the projection of the image of the sun, the sun itself being supposed circular, may be regarded as a circle drawn round the projection of the sun's center as a center. Indeed, it is easy to see, by geometrical construction, that the variations in the radius of the projected image will be of the order of magnitude of $\beta_0^2 a'' f$, which we have neglected throughout.

If we denote by ω the angle of position on the photographic plate, reckoned from the sun's center, putting

$$\left. \begin{aligned} \eta_1 &= \rho \cos \omega \\ \zeta_1 &= -\rho \sin \omega \end{aligned} \right.$$

we have for the position-angle ω ,

$$\tan \omega = -\frac{\zeta_1}{\eta_1} = -\frac{\gamma_0 - \beta_0 a'' \tan \delta_0}{\beta_0 + \gamma_0 a'' \tan \delta_0}$$

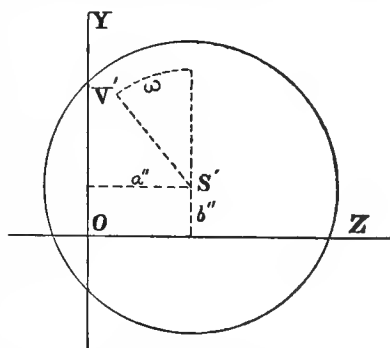
which, being compared with (21), shows that, neglecting quantities of the fourth order, we have

$$\omega = p - A + a'' \tan \delta_0 \} \dots \dots (27)$$

It will be seen that the quantities $a, b, a',$ and b' do not individually appear in the expression for the relative co-ordinates of Venus and the sun.

10. We now reduce the formulæ already found to a form for computing the angle of position and distance of Venus from the sun's center, from the measures on the photographic plate. It is assumed that the photographic telescope is so near the meridian, and so nearly horizontal, that the deviation of the reflected central ray from the sun's center, from a horizontal meridian line (the arc $O S'$ in the figure), does not in general exceed $6'$ of arc.

The figure is supposed to represent the photographic plate as seen from the objective, the line $O Y$ to represent the true meridian and therefore to be vertical, and $O Z$ to be truly horizontal and on a level with the optical center of the objective. For the southern hemisphere, the figure must be turned upside down, so that the positive axis of Y shall point downward, and that of Z to the left.



It is assumed that, by a suitable micrometer, the co-ordinates of the center of Venus or of V' , relative to the center of the sun, or of S' , are determined with all possible accuracy, so that we know the distance $S' V'$, which

we call ρ , in millimeters or other linear measure, and the angle which the line $S'V'$ makes with the vertical line OY , which angle, counted in the direction of the hands of a watch, from OY toward OZ , we represent by ω . If the actual ruled lines on the plate are not truly horizontal and vertical, the value of ω must be corrected for their inclination, and if the plate has no fiducial lines exactly in the meridian, the azimuth and elevation of the sun's center must be corrected for the azimuth and elevation of the fiducial lines, so that we shall know the azimuth a'' and the elevation b'' to the nearest tenth of a minute of arc for each picture. It is also assumed that the distance from the center of divergence of the objective, or, if the mirror has any curvature, the distance from the mean optical center of the combination of mirror and objective, to the center of the photographic plate, is known in the same linear measure with which $S'V'$ is determined. This distance is called f . We also put

δ_0 , the declination of the horizon toward which the sun's rays are reflected;
 H, δ , the apparent hour angle (west) and declination of the sun's center, as seen from the station, affected by parallax and refraction, to the nearest tenth of a minute of arc. These quantities may be computed by correcting the local mean time by the equation of time and then applying parallax and refraction in R. A. and Dec. The quantities

$$\omega, \rho, a'', b'', \delta_0, \delta, H, f,$$

are thus assumed to be known. Now compute

$$\delta'' = \delta_0 + b''$$

$$\tan \frac{1}{2} A = \frac{\cos \frac{1}{2}(\delta - \delta'')}{\sin \frac{1}{2}(\delta + \delta'')} \cot \frac{1}{2}(H - a'' \sec \delta_0) \dots \dots \dots (28)$$

$$p = A + \omega - a'' \tan \delta_0 \dots \dots \dots (29)$$

p is the angle of position of Venus relative to the sun's center and the north point of his limb, as it appears from the station. It must be next corrected for the relative refraction of Venus and the sun.

The apparent angular distance of the centers of Venus and the sun is given by the equation

$$\tan s = \frac{\rho}{f} \dots \dots \dots (30)$$

It is also to be corrected for relative refraction.

11. Instead of taking the right ascension and declination of the sun as our fundamental quantities, we might equally well have used altitudes and azimuths. The actual formulæ to be finally employed in the computation would have been substantially the same, except that in computing A we should have put the sun's altitude in place of δ and its azimuth in place of $H - a'' \sec \delta_0$. The value of p would then have been referred, not to the meridian passing through the sun, but to the vertical circle passing through it.

§ 4. CORRECTIONS OF THE OBSERVED POSITION OF VENUS ON ACCOUNT OF REFRACTION.

In the preceding investigation we have supposed the position of the sun, as seen from the station, at the moment of taking the photograph, to be affected by refraction, and we have shown how, when this position is computed, we may deduce, from the measures on the photographic plate, the position-angle and distance of the center of Venus, reckoned from that of the sun, as they must have appeared at the station. These relative co-ordinates are now to be corrected for refraction in order that we may obtain their values free from refraction. It is first to be remarked that what we really obtain from the measures on the plate is not the position of Venus relative to the true center of the sun considered as a point, but relative to a number of points on the sun's limb, from which the position relative to the center is to be deduced. We have, therefore, to inquire into the effects of refraction upon the relative position of the sun's center and any point upon its limb—a problem which is nearly identical with that of determining the effect of the same cause upon the relative positions of the centers of the sun and Venus. This problem will be treated after the manner of BESSEL, except that, instead of the angle which the middle point of the line joining the two centres makes with the meridian or the vertical circle, we shall consider the angle at the center of the sun, and shall put the results into a form more convenient for our present purpose.

1. The quantities on which the various effects of refraction depend, such as the zenith distance and semi-diameter of the sun, and the parallactic angle, are to be computed from the tables. They are, in general, only required to be accurate to tenths of minutes; it is therefore indifferent what tables we use in the computation. Let us put

\mathcal{Z} , the zenith distance of the sun's center;

ρ , the change of \mathcal{Z} due to refraction;

q , the parallactic angle corresponding to the sun's center; or the angle of position of the sun's vertex counted from his north point toward the left;

s , the angular distance of Venus, or of any point on the sun's limb, from the sun's center. We may call this simply "the point";

V , the angle of position of the point, referred to the sun's center, and counted from the vertex toward the left;

R , the angle between the vertical circles drawn through the sun's center, and through the point, respectively;

H , the west hour angle as seen from the station.

The values of \mathcal{Z} , ρ , &c., which refer to Venus or the second point, will be distinguished by a subscript index, or, as \mathcal{Z}_v , ρ_v , &c. V_1 is the angle which the great circle from the sun's center through the point makes with the vertical circle through the point.

Where it is necessary to distinguish between the values of the above quantities, as affected by refraction and as freed from it, we shall use two accents to mark those quantities which are affected by refraction. The unaccented quantities will therefore

be those computed from the tables without refraction, while z'' , q'' , &c., will be the apparent values of the corresponding quantities affected by refraction.

We shall, when necessary, designate by a single accent mean values between those affected by and freed from refraction.

2. The relations which connect the quantities S , V , and V_1 with z , z_1 , and R , are

$$\left. \begin{aligned} \cos s &= \cos z \cos z_1 + \sin z \sin z_1 \cos R \\ \sin s \sin V &= \sin R \sin z_1 \\ \sin s \cos V &= \sin z \cos z_1 - \cos z \sin z_1 \cos R \\ \sin s \sin V_1 &= \sin R \sin z \\ \sin s \cos V_1 &= \cos z \sin z_1 - \sin z \cos z_1 \cos R \end{aligned} \right\} \dots (31)$$

We may consider the same relations to hold true when s , V , z , and z_1 are affected with accents.

If we differentiate the first of these equations with respect to z and z_1 , and then substitute in the second member the values of z , z_1 , etc., from the other equations, we find

$$ds = \cos V dz - \cos V_1 dz_1$$

If we represent by the symbol δ the corrections to be applied to the refracted values of the various quantities to obtain the true values, we shall have, with all necessary accuracy,

$$\delta s = \cos V' \delta z - \cos V_1' \delta z_1 \dots (32)$$

If we put Δ to represent the difference between the quantities referring to Venus and those referring to the sun's center, we shall have,

$$\Delta \rho = \rho_1 - \rho = \delta z_1 - \delta z$$

and thus we shall have

$$\delta s = \rho (\cos V' - \cos V_1') - \Delta \rho \cos V_1' \dots (33)$$

The greatest zenith distance at which the sun was advantageously photographed during the late transit is probably less than 79° . We shall then have, in round numbers,

$$\begin{aligned} \text{Max. } \rho &= 300'' \\ \text{Max. } \Delta \rho &= 6'' \end{aligned}$$

Therefore, to obtain δs to the accuracy of $0''.01$, we shall require $\cos V - \cos V_1$ to $0'.1$ and V_1 to $5'$. From the equation

$$\cos V_1 = \cos R \cos V - \sin R \sin V \cos z$$

we obtain

$$\cos V - \cos V_1 = 2 \sin^2 \frac{1}{2} R \cos V + \sin R \sin V \cos z.$$

The maximum value of $2 \sin^2 \frac{1}{2} R \cos V$ for zenith distances exceeding 60° is less

than .00001; its product by ρ may therefore be entirely neglected, unless the sun is in the immediate neighborhood of the zenith. We may therefore put

$$\begin{aligned} \cos V - \cos V_1 &= \sin R \sin V \cos \mathcal{Z} \\ &= \frac{\sin s \sin^2 V \cos \mathcal{Z}}{\sin \mathcal{Z}_1} \end{aligned}$$

since the spherical triangle of which the sides are s , \mathcal{Z} , and \mathcal{Z}_1 gives

$$\frac{\sin R}{\sin s} = \frac{\sin V}{\sin \mathcal{Z}_1}$$

The maximum value of $\rho \sin s$ being little more than 1'', the factor $\frac{\cos \mathcal{Z}}{\sin \mathcal{Z}_1}$ need be true only to its hundredth part. We may therefore put

$$\mathcal{Z}_1 = \mathcal{Z} - \sin s \cos V$$

From this we obtain, by developing to quantities of the first order in s ,

$$\frac{1}{\sin \mathcal{Z}_1} = \frac{1 + \sin s \cos V \cot \mathcal{Z}}{\sin \mathcal{Z}}$$

which, being substituted in the preceding value of $\cos V - \cos V_1$, gives

$$\cos V - \cos V_1 = \sin s \sin^2 V \cot \mathcal{Z} (1 + \sin s \cos V \cot \mathcal{Z})$$

an expression in which the last factor may be put equal to unity for all zenith distances greater than 45° .

These same equations will hold substantially true if we affect all the quantities which enter into them by one or two accents. We have therefore, for the first term of δs in (33),

$$\rho (\cos V' - \cos V_1') = \rho \sin s' \sin^2 V' \cot \mathcal{Z}' (1 + \sin s' \cos V' \cot \mathcal{Z}') . \quad (34)$$

Passing now to the second term, we remark that the value of the expression

$$\Delta \rho (\cos V - \cos V_1)$$

is always less than 0''.002 at all zenith distances at which the sun was photographed during the transit of Venus. We may therefore put $\Delta \rho \cos V$, instead of $\Delta \rho \cos V_1$, in the second term of δs . We have now to find $\Delta \rho$ from the refraction tables as a function of the zenith distance. This value can be readily expressed in the form

$$\Delta \rho = \rho \{n_1 (\mathcal{Z}_1 - \mathcal{Z}) + n_2 (\mathcal{Z}_1 - \mathcal{Z})^2\}, \quad \quad (35)$$

where n_1 and n_2 may, with all necessary accuracy, be regarded as a function of the zenith distance simply, and tabulated as such. Moreover, the values of n_1 and n_2 can be so chosen that \mathcal{Z} may represent either \mathcal{Z} , \mathcal{Z}' or \mathcal{Z}'' .

We have, to terms of the second order in s ,

$$\mathcal{Z}_1 - \mathcal{Z} = -s \cos V + \frac{1}{2} s \sin s \cot \mathcal{Z} \sin^2 V$$

The second term may be omitted entirely since, at the zenith distance of 80° , it only amounts to about $0''.5$. Substituting the first term in the expression for $\Delta \rho$, we find

$$-\Delta \rho \cos V = s \rho \{n_1 \cos^2 V - n_2 \sin s \cos^3 V\} \dots (36)$$

The sum of (34) and (36) will be the total correction for refraction in distance. We can reduce it to a more convenient form after we find the corresponding correction to the angle V .

3. If we differentiate the second and third of the equations (31), we find

$$\begin{aligned} \sin s \cos V dV + \cos s \sin V ds &= \sin R \cos \mathcal{Z}_1 d\mathcal{Z}_1 \\ -\sin s \sin V dV + \cos s \cos V ds &= \cos s d\mathcal{Z} \\ &- (\sin \mathcal{Z} \sin \mathcal{Z}_1 + \cos \mathcal{Z} \cos \mathcal{Z}_1 \cos R) d\mathcal{Z}_1. \end{aligned}$$

Multiplying the first of these equations by $\cos V$, and the second by $-\sin V$, and adding, noticing also that

$$\begin{aligned} \sin R \cos V + \cos R \sin V \cos \mathcal{Z} &= \sin V_1 \cos \mathcal{Z}_1 \\ \sin V \sin \mathcal{Z} &= \sin V_1 \sin \mathcal{Z}_1 \end{aligned}$$

we find

$$\sin s dV = -\cos s \sin V d\mathcal{Z} + \sin V_1 d\mathcal{Z}_1$$

Putting, as before, the refractions for the differentials of the zenith distances, we have, with all necessary accuracy,

$$\sin s' \delta V = \rho (\sin V_1' - \cos s' \sin V') + \Delta \rho \sin V_1'$$

From the fundamental formulæ of spherical trigonometry, we have

$$\cos s \sin V_1 = \cos R \sin V + \sin R \cos V \cos \mathcal{Z}$$

and thence

$$\sin V_1' - \cos s' \sin V' = (\cos R \sec s' - \cos s') \sin V' + \sin R \cos V' \sec s' \cos \mathcal{Z}'$$

For considerable zenith distances, the first term of the second member of this equation will never amount to $.00002$; its product by ρ may therefore be neglected. The second term may be reduced to

$$\tan s' \sin V_1' \cos V' \cot \mathcal{Z}'$$

For zenith distances exceeding 60° , the value of $\sin V - \sin V_1$ will never exceed $.001$; we may therefore put V' for V_1' in the last expression, from which the first term in the above value of $\sin s' \delta V$ will become

$$\rho \tan s' \sin V' \cos V' \cot \mathcal{Z}'$$

For the same reason as in the case of δs we may put V' for V_1' in the second

term of $\sin s' \delta V$, so that we shall have, by substituting the value of $\Delta \rho$ already found, reducing and neglecting the difference between $\sin s$ and $\tan s$

$$\left. \begin{aligned} \sin s' \delta V &= \rho \left\{ \sin s' \sin V' \cos V' \cot \mathcal{Z}' \right. \\ &\quad \left. - \sin s' (n'_1 \cos V' \sin V' - n'_2 \cos^2 V' \sin V') \right\} \\ \delta V &= \rho \sin V' \left\{ \cos V' \cot \mathcal{Z}' - n'_1 \cos V' + n'_2 s \cos^2 V' \right\} \end{aligned} \right\} . \quad (37)$$

In the case of δV it is a matter of indifference whether we express it as a function of the unaccented quantities or of either of the accented ones, and the same remark will apply to δs when s represents the angular distance of the centers of the Sun and Venus. But in the case of the Sun's angular radius it will be more convenient to have a constant s for the various measures than one varying with each measure; we must therefore express δs for this particular case as a function of s unaccented. Putting s for $\sin s$, and supposing, also, that we take n_1 and n_2 as functions of the singly accented zenith distances, the value of δs , as we have found it from the sum of (32) and (33), in the preceding section, will be

$$\delta s = \rho s' \left\{ n'_1 \cos^2 V' + \cot \mathcal{Z}' \sin^2 V' + s' \cos V' (\cot^2 \mathcal{Z}' \sin^2 V' - n'_2 \cos^2 V') \right\} . \quad (38)$$

To express this in terms of s we have only to put $s - \frac{1}{2} \delta s$ for s' in the second member of the equation, while for the latter value of δs it will be sufficient to suppose

$$\frac{1}{2} \delta s = \frac{1}{2} \rho s n'_1 \cos^2 V'$$

This being substituted in the expression for s' outside the parenthesis, the terms added to δs will be

$$-\frac{1}{2} \delta s \times \rho n'_1 \cos^2 V' = -\frac{1}{2} \rho^2 s n'^2_1 \cos^2 V'$$

Owing to the minuteness of the terms multiplied by s'^2 it will be indifferent whether we use s or s' inside the parenthesis; the total expressions for δs will therefore be

$$\left. \begin{aligned} \delta s &= \rho s \left\{ n'_1 \cos^2 V' + \cot \mathcal{Z}' \sin^2 V' - \frac{1}{2} \rho n'^2_1 \cos^2 V' \right. \\ &\quad \left. + s \cos V' (\cot^2 \mathcal{Z}' \sin^2 V' - n'_2 \cos^2 V') \right\} \end{aligned} \right\} . \quad (39)$$

4. In measuring the photographs, the measures are always made along two nearly opposite radii; the position of the sun's center will depend solely upon the difference of the radii and the sun's diameter upon the sum. Since the former quantity is the important one in the determination of the position of Venus upon the sun's face, while the latter is entirely subordinate, it will be better to obtain the sum and difference of δs for two opposite points. Let us then suppose

$$V' = \alpha; \quad V' = \alpha + 180^\circ$$

to be two opposite values of V' to be substituted in the expression for δs , and let us represent by δs_1 and δs_2 the corresponding values of δs . We find

$$\frac{1}{2} (\delta s_1 - \delta s_2) = \rho s^2 \cos \alpha (\cot^2 \mathcal{Z}' \sin^2 \alpha - n'_2 \cos^2 \alpha)$$

The values of ρ and s^2 are, approximately,

$$\begin{aligned}\rho &= 60'' \tan \mathcal{Z}'_1 \\ s^2 &= .00002\end{aligned}$$

giving for the approximate value of the first term of the expression sought

$$0''.0012 \cot \mathcal{Z} \cos \alpha \sin^2 \alpha,$$

which may be entirely neglected. The value of the expression for the displacement of the sun's center reduces to

$$\frac{1}{2} (\delta s_1 - \delta s_2) = -\rho n'_2 s^2 \cos^3 \alpha$$

For the correction of the semi-diameter, or of half the measured diameter, we have

$$\frac{1}{2} (\delta s_1 + \delta s_2) = \rho s \{n'_1 \cos^2 \alpha + \cot \mathcal{Z}' \sin^2 \alpha - \frac{1}{2} \rho n'^2_1 \cos^4 \alpha\} \quad . \quad (40)$$

The greatest effect of the last term at zenith distance, 79° , is only $0''.05$, and it diminishes very rapidly with the zenith distance. It may therefore be entirely neglected in obtaining the sun's semi-diameter from the photographs.

The actual measures of the photographs have been made at twelve equidistant points around the limb of the sun. There will therefore be twelve equidistant values of V and six of α to be taken into consideration, the latter being all in one-half of the circle. The position of the sun's center in the direction of the vertical co-ordinate is given as a function of the readings around the sun's circumference in the form

$$h = \frac{\sum s_i \cos V_i}{\sum \cos^2 V_i} = \frac{2}{n} \sum s_i \cos V_i$$

s being the reading in the direction of the sun's radius, n the number of readings, here 12, and i the index by which they are distinguished. If now we determine the position of the center without regard to refraction, the total correction on account of refraction will be

$$\frac{1}{3} \rho n'_2 s^2 \sum \cos^4 \alpha_i$$

Taking for the six values of α_i the values of V_i , which are on one side of a horizontal diameter, we shall have, very nearly,

$$\frac{1}{3} \sum \cos^4 \alpha_i = 0.75$$

so that the correction to be applied to the zenith distance of the sun's center for refraction is

$$\delta \mathcal{Z} = 0.75 \rho n'_2 s^2$$

This is the quantity by which the zenith distance of the sun's true center exceeds that of the center of his apparent mean circumference in consequence of the differential refraction. It may be tabulated as a function of \mathcal{Z} .

The correction to the semi-diameter may be reduced to a similar form.

5. Since the error of the tabular place of Venus from Hill's tables cannot be more than $3''$ in arc of a great circle, and is probably less than $2''$, it seems better to express the corrections for the effect of refraction upon the position of Venus in terms of the singly accented quantities, and to take as the values of these quantities with which to enter the tables the mean of the tabular values, and of those derived immediately from the photographic measures. The values of the corrections sought will then be

$$\begin{aligned} \delta s &= \rho s' \{n'_1 \cos^2 V' + \cot \mathcal{Z}' \sin^2 V' + s \cos V' (\cot^2 \mathcal{Z} \sin^2 V' - n'_2 \cos^2 V')\} \\ \delta V &= \rho \sin V' \cos V' \{ \cot \mathcal{Z}' - n'_1 + n'_2 s \cos V' \} \end{aligned}$$

The position-angle from the north point, which we have represented by p , is given by the equation

$$p = V + q.$$

The parallactic angle q is affected by refraction as well as V . The hour angle and declination of the sun and the latitude of the place are supposed to be known, and from them the sun's parallactic angle, zenith distance, and azimuth may be determined, the six quantities in question, or their complements or supplements, forming the parts of a spherical triangle, of which the angles are at the pole, the zenith, and the direction of the sun's center. The equations for determining the quantities q , \mathcal{Z} , and azimuth, a may be expressed in the form

$$\begin{aligned} \sin \mathcal{Z} \sin q &= \cos \varphi \sin H \\ \sin \mathcal{Z} \cos q &= \sin \varphi \cos \delta - \cos \varphi \sin \delta \cos H \\ \cos \mathcal{Z} &= \sin \varphi \sin \delta + \cos \varphi \cos \delta \cos H \\ \sin \mathcal{Z} \sin a &= \cos \delta \sin H \\ \sin \mathcal{Z} \cos a &= -\cos \varphi \sin \delta + \sin \varphi \cos \delta \cos H \end{aligned}$$

If all three quantities, a , \mathcal{Z} , and q are required, they may also be obtained from Gauss's equations,

$$\begin{aligned} \sin \frac{1}{2}(a + q) \sin \frac{1}{2} \mathcal{Z} &= \sin \frac{1}{2} H \cos \frac{1}{2}(\varphi + \delta) \\ \cos \frac{1}{2}(a + q) \sin \frac{1}{2} \mathcal{Z} &= \cos \frac{1}{2} H \sin \frac{1}{2}(\varphi - \delta) \\ \sin \frac{1}{2}(a - q) \cos \frac{1}{2} \mathcal{Z} &= \sin \frac{1}{2} H \sin \frac{1}{2}(\varphi + \delta) \\ \cos \frac{1}{2}(a - q) \cos \frac{1}{2} \mathcal{Z} &= \cos \frac{1}{2} H \cos \frac{1}{2}(\varphi - \delta) \end{aligned}$$

Actually, however, the computation of a may be dispensed with, and then the first three equations will be the most convenient. They may be a little simplified by the following process: Find two auxiliaries, h and ψ , from the equations

$$\begin{aligned} k \sin \psi &= \sin \varphi \\ k \cos \psi &= \cos \varphi \cos H \end{aligned}$$

we shall then have,

$$\begin{aligned} \sin \mathcal{Z} \sin q &= \cos \varphi \sin H \\ \sin \mathcal{Z} \cos q &= k \sin(\psi - \delta) \\ \cos \mathcal{Z} &= k \cos(\psi - \delta) \end{aligned}$$

The two quantities $\sin \mathcal{Z}$ and $\cos \mathcal{Z}$, being formed independently, will serve as a check upon the computation.

Considering the right ascension, declination, and parallactic angle as functions of the azimuth and zenith distance, the differential co-efficients of the quantities in question with respect to \mathcal{Z} are

$$\frac{d H}{d \mathcal{Z}} = \sin q \sec \delta$$

$$\frac{d \delta}{d \mathcal{Z}} = -\cos q$$

$$\frac{d q}{d \mathcal{Z}} = -\sin q \tan \delta$$

From the last of these equations we derive

$$q'' - q = -\delta q = \rho \sin q \tan \delta.$$

For the correction to free the observed position-angle p from the effects of refraction, we have

$$\begin{aligned} p - p'' &= V - V'' + q - q'' \\ &= \frac{1}{2} \rho \sin 2 V' \{ \cot \mathcal{Z}' - n'_1 + n'_2 s \cos V' \} - \rho \sin q \tan \delta \end{aligned}$$

6. It remains to tabulate the numerical data for applying the corrections for refraction, beginning with the quantities n_1 and n_2 . In doing this, we shall make use of the Pulkowa refraction tables.* The formula here used for the refraction, so far as it is necessary for our present purpose, is of the form

$$\log \rho = m + A (B + T) + \lambda \gamma$$

m , A and λ being functions of the zenith distance, while B , T and γ are the factors of corrections for the barometer and thermometers. Differentiating this expression twice, the results may be expressed in the form

$$\frac{d \rho}{d \mathcal{Z}} = \rho N_1$$

$$\frac{d^2 \rho}{d \mathcal{Z}^2} = \rho N_2$$

where $M =$ modulus of logarithms 0.4343

$$N_1 = \frac{1}{M} \left\{ \frac{d m}{d \mathcal{Z}} + (B + T) \frac{d A}{d \mathcal{Z}} + \gamma \frac{d \lambda}{d \mathcal{Z}} \right\}$$

$$N_2 = N_1^2 + \frac{d N_1}{d \mathcal{Z}}$$

* Tabulæ Refractionum in usum Speculæ Pulkovensis congestæ. Petropoli, 1870.

Owing to the extreme minuteness of the factors $\frac{d A}{d \mathcal{E}}$ and $\frac{d \lambda}{d \mathcal{E}}$, and the generally small values of the corrections B, T, and γ , we may neglect the products of these quantities. We shall then have

$$N_1 = \frac{1}{M} \frac{d m}{d \mathcal{E}} = 2.30 \frac{d m}{d \mathcal{E}}$$

$$N_2 = \frac{1}{M^2} \frac{d m^2}{d \mathcal{E}^2} + \frac{1}{M} \frac{d^2 m}{d \mathcal{E}^2} = N_1^2 + 2.30 \frac{d^2 m}{d \mathcal{E}^2}$$

The expression for $\Delta \rho$ being

$$\Delta \rho = (\mathcal{E}_1 - \mathcal{E}) \frac{d \rho}{d \mathcal{E}} + \frac{1}{2} (\mathcal{E}_1 - \mathcal{E})^2 \frac{d^2 \rho}{d \mathcal{E}^2}$$

we find, by comparing with (35)

$$n_1 = N_1$$

$$n_2 = \frac{1}{2} N_2$$

so that the values of n_1 and n_2 are

$$n_1 = 2.30 \frac{d m}{d \mathcal{E}}$$

$$n_2 = \frac{1}{2} n_1^2 + 1.15 \frac{d^2 m}{d \mathcal{E}^2}$$

The values of m , etc., which are to be used in these expressions, are those corresponding to the case in which the argument is the mean of the true and apparent zenith distances of the sun, and in the table the formulæ have been made to correspond to this case. As a matter of fact, the difference between the results obtained by taking the one zenith distance or the other as the independent variable may be regarded as unimportant.

The various quantities needed for the corrections on account of refraction, so far as they are a function of the zenith distance, are given in the following table. There (ρ) represents the mean refraction, π the parallax in altitude, and $\Delta \mathcal{E}$ their sum, as already used in the formulæ. The argument is supposed to be \mathcal{E}' , or the mean of the zenith distances as corrected and as uncorrected for refraction.

The value of ρ to be used in computing δs and δV should be computed from the refraction tables, having regard to the readings of the thermometer and barometer.

TABLE FOR REFRACTION IN POSITION ANGLE AND DISTANCE.

ζ'	(ρ)	π	$\Delta\zeta$	$\cot \zeta'$	n'_1	$n'_1 - \cot \zeta'$	ζ'	(ρ)	π	$\Delta\zeta$	$\cot \zeta'$	n'_1	$n'_1 - \cot \zeta'$
0	"	"	"				0	"	"	"			
20	+21.0	-3.1	+18	2.75	3.11	+0.36	40	48.4	-5.8	43	1.19	2.03	0.83
21	22.2	-3.2	19	2.61	3.00	.38	41	50.2	-5.9	44	1.15	2.02	.86
22	23.3	-3.4	20	2.48	2.88	.40	42	51.9	-6.0	46	1.11	2.02	.90
23	24.5	-3.6	21	2.36	2.78	.42	43	53.8	-6.1	48	1.07	2.01	.93
24	25.7	-3.7	22	2.25	2.69	.45	44	55.7	-6.2	49	1.04	2.00	.96
25	26.9	-3.8	23	2.14	2.61	.47	45	57.7	-6.4	51	1.00	2.00	0.99
26	28.2	-3.9	24	2.05	2.54	.49	46	59.7	-6.5	53	0.966	2.00	1.03
27	29.4	-4.1	25	1.96	2.47	.51	47	61.8	-6.6	55	0.933	2.00	1.07
28	30.7	-4.2	26	1.88	2.41	.53	48	64.0	-6.7	57	0.900	2.01	1.11
29	32.0	-4.4	28	1.80	2.36	.55	49	66.3	-6.8	60	0.869	2.02	1.15
30	33.3	-4.5	29	1.73	2.31	.58	50	68.7	-6.9	62	0.839	2.03	1.19
31	34.7	-4.6	30	1.66	2.27	.60	51	71.2	-7.0	64	0.810	2.03	1.22
32	36.1	-4.8	31	1.60	2.23	.62	52	73.8	-7.1	67	0.781	2.05	1.27
33	37.5	-4.9	33	1.54	2.20	.65	53	76.5	-7.2	69	0.754	2.07	1.31
34	38.9	-5.0	34	1.48	2.16	.67	54	79.3	-7.3	72	0.727	2.09	1.36
35	40.4	-5.2	35	1.43	2.13	.70	55	82.3	-7.4	75	0.700	2.12	1.42
36	41.9	-5.3	37	1.38	2.10	.72	56	85.4	-7.5	78	0.675	2.14	1.47
37	43.5	-5.4	38	1.33	2.08	.75	57	88.7	-7.5	81	0.649	2.17	1.52
38	45.1	-5.5	40	1.28	2.06	.78	58	92.1	-7.6	85	0.625	2.21	1.58
39	46.7	-5.7	41	1.23	2.04	.80	59	95.8	-7.7	88	0.601	2.25	1.64
40	+48.4	-5.8	+43	1.19	2.03	0.83	60	99.7	-7.8	92	0.577	2.29	1.71

ζ'	(ρ)	π	$\Delta\zeta$	$\cot \zeta'$	n'_1	$n'_1 - \cot \zeta'$	n'_2	$\delta'' \zeta$	$\frac{\rho n'_2}{100}$	$\log \rho n'_2$
0	"	"	"					"		
60	1 39.7	-7.8	1 32	0.587	2.292	1.71	4.0	0.01	4	2.60
61	1 43.8	-7.9	1 36	0.554	2.338	1.78	4.2	.01	4	2.63
62	1 48.2	-7.9	1 40	0.532	2.390	1.86	4.5	.01	5	2.68
63	1 52.8	-8.0	1 45	0.510	2.443	1.93	4.8	.01	5	2.73
64	1 57.8	-8.1	1 50	0.488	2.517	2.03	5.1	.01	6	2.78
65	2 3.2	-8.2	1 55	0.466	2.581	2.12	5.4	.01	7	2.82
66	2 8.9	-8.2	2 1	0.445	2.657	2.21	5.8	.01	7	2.87
67	2 15.2	-8.3	2 7	0.425	2.738	2.31	6.2	.01	8	2.92
68	2 21.9	-8.3	2 14	0.404	2.831	2.43	6.8	.02	10	2.98
69	2 29.3	-8.4	2 21	0.384	2.940	2.56	7.6	.02	11	3.05
70	2 37.3	-8.5	2 29	0.364	3.052	2.69	8.3	.02	13	3.12
71	2 46.1	-8.5	2 38	0.344	3.178	2.83	9.1	.02	15	3.18
72	2 55.8	-8.6	2 47	0.325	3.320	2.99	10.0	.03	18	3.24
73	3 6.6	-8.6	2 58	0.306	3.478	3.17	11.2	.03	21	3.31
74	3 18.6	-8.7	3 10	0.287	3.661	3.37	12.3	.04	24	3.39
75	3 32.1	-8.7	3 23	0.268	3.865	3.60	13.6	.05	29	3.46
76	3 47.4	-8.7	3 39	0.249	4.100	3.85	15.5	.06	35	3.54
77	4 49	-8.8	3 56	0.231	4.355	4.12	17.5	.07	43	3.63
78	4 25.0	-8.8	4 16	0.213	4.658	4.44	20.1	.09	53	3.73
79	4 48.5	-8.8	4 40	0.194	5.014	4.82	23.5	.11	67	3.83
80	5 16.2	-8.9	5 7	0.176	5.423	5.25	27.3	.14	86	3.94

§ 5. EXPRESSION OF THE POSITION OF VENUS ON THE SUN'S DISC IN TERMS OF TABULAR ELEMENTS.

The object of what precedes is to determine, from the measures on the photographic plate, the position angle and distance of the center of Venus from that of the Sun as they would have been seen from the station, were there no refraction. These quantities will form the right-hand members of equations of condition on Professor AIRY'S system of reduction. The left-hand members will be the corresponding quantities computed from theory, and containing symbolic corrections to the doubtful elements of the theory. The formation of the latter quantities will next claim our attention. The methods of doing this are so simple and well understood that it is not necessary to discuss them in detail, and, since only one result can be correctly arrived at, it is merely a question of convenience to the computer whether one or another method shall be adopted.

Since the distances and position angles are found from observation by entirely different instrumentalities, each subject to its own sources of error, it is necessary to discuss them separately. We must therefore find, for the moment of each photograph, the theoretical position-angle and distance of Venus from the Sun's center, as affected by arbitrary corrections to the elements, and as seen from the station. The method of doing this which seems to offer most security against error is to prepare a table of the geocentric values of s and p , and to compute the effects of parallax upon these quantities for each observation and each station. The system on which this is done is substantially that of OPPOLZER in the *Sitzungsberichte der Wiener Akademie*, vol. 61, p. 515, Vienna, 1870. The following is a brief outline of the way in which the formulæ have been derived:

First, s and p are given with sufficient accuracy by the equations

$$\left. \begin{aligned} s \sin p &= \cos \delta_1 (\alpha_1 - \alpha) = A \cos \delta_1 \\ s \cos p &= \delta_1 - \delta + \frac{1}{4} A \sin A \sin 2 \delta \\ &= \delta_1 - \delta - [9.252] A \sin A \end{aligned} \right\} \dots \dots (43)$$

A being the difference of right ascensions and α_1 and δ_1 the R. A. and Dec. of Venus. In these formulæ quantities of the third order with respect to s are neglected; that is, the sines of $\alpha_1 - \alpha$ and $\delta_1 - \delta$ are taken the same as the arcs themselves

In determining the effect of parallax, quantities of the second order with respect to the parallax may be entirely neglected except when they contain quantities of the order of magnitude of s as a divisor. We put

- π , the equatorial horizontal parallax of the Sun for the time of observation;
- π_1 , that of Venus;
- H , the west hour angle of the Sun;
- $h = \rho \sin \varphi'$ } ρ being here the earth's radius at the station and φ' the geocen-
- $k = \rho \cos \varphi'$ } tric latitude;
- $A = \alpha_1 - \alpha = H - H_1$;
- Δ , symbol for change produced by parallax.

By differentiating the expressions for $s \sin p$ and $s \cos p$, and using $\sin A$ instead of A , we obtain

$$\begin{aligned}\Delta (s \sin p) &= \cos \delta_1 \Delta A - \sin A \sin \delta_1 \Delta \delta_1 \\ \Delta (s \cos p) &= \Delta \delta_1 - \Delta \delta + \frac{1}{2} \sin A \sin 2 \delta \Delta A\end{aligned}$$

We have, by well known formulæ for the parallax in right ascension and declination

$$\begin{aligned}\Delta A &= k (\pi \sec \delta \sin H - \pi_1 \sec \delta_1 \sin H_1) \\ \Delta \delta &= k \pi \sin \delta \cos H - h \pi \cos \delta \\ \Delta \delta_1 &= k \pi_1 \sin \delta_1 \cos H_1 - h \pi_1 \cos \delta_1\end{aligned}$$

If we consider only terms of the first order in A and $\delta_1 - \delta$, and therefore put

$$\begin{aligned}\sin H_1 &= \sin H - \sin A \cos H \\ \cos H_1 &= \cos H + \sin A \sin H\end{aligned}$$

then, by suitable reductions, by putting, for brevity,

$$\begin{aligned}m &= \pi \cos \delta_1 \sec \delta - \pi_1 \\ n &= \pi_1 \sin A \cos^2 \delta_1 \\ m' &= \pi_1 \sin \delta_1 - \pi \sin \delta \\ n' &= \pi \sin A \sin \delta \\ p &= \frac{1}{2} \pi_1 \sin A \sin 2 \delta_1 \\ p' &= \pi \cos \delta - \pi_1 \cos \delta_1\end{aligned}$$

and by neglecting quantities of the second order in A and $\delta_1 - \delta$, we find

$$\left. \begin{aligned}\Delta (s \sin p) &= k (m \sin H + n \cos H) + h p \\ \Delta (s \cos p) &= k (m' \cos H + n' \sin H) + h p'\end{aligned} \right\} \dots (44)$$

It will be seen that m, n, p, m' , etc., are functions of the absolute time alone, which can be tabulated as such, while k and h are constants for each station, leaving H , the Sun's hour angle, as the only independent variable at any one station.

If, now, we put, with OPPOLZER,

$$\left. \begin{aligned}\Delta_1 &= \cos p_0 \Delta (s \sin p) - \sin p_0 \Delta (s \cos p) \\ \Delta_2 &= \sin p_0 \Delta (s \sin p) + \cos p_0 \Delta (s \cos p)\end{aligned} \right\} \dots (45)$$

a subscript zero designating geocentric quantities, so that we have

$$\begin{aligned}\Delta (s \sin p) &= s \sin p - s_0 \sin p_0 \\ \Delta (s \cos p) &= s \cos p - s_0 \cos p_0\end{aligned}$$

we find

$$\left. \begin{aligned}s \sin (p - p_0) &= \Delta_1 \\ s \cos (p - p_0) &= s_0 + \Delta_2\end{aligned} \right\} \dots (46)$$

from which s and $p - p_0$ can be obtained either by a rigorous computation or by the following approximation,

$$\begin{aligned}\Delta p = p - p_0 &= \frac{\Delta_1}{\sin s_0} \left(1 - \frac{\Delta_2}{s_0} \right) \\ \Delta s = s - s_0 &= \Delta_2 + \frac{s_0}{2} (\Delta p \sin 1'')^2\end{aligned}$$

We have now to consider the most convenient expressions for Δ_1 and Δ_2 . By substituting (44) in (45), we have

$$\begin{aligned}\Delta_1 &= k \{ (m \cos p_0 - n' \sin p_0) \sin H + (n \cos p_0 - m' \sin p_0) \cos H \} \\ &\quad + h (p \cos p_0 - p' \sin p_0) \\ \Delta_2 &= k \{ (m \sin p_0 + n' \cos p_0) \sin H + (n \sin p_0 + m' \cos p_0) \cos H \} \\ &\quad + h (p \sin p_0 + p' \cos p_0)\end{aligned}$$

If we put

$$\begin{aligned}R \cos \theta &= m \cos p_0 - n' \sin p_0 \\ R \sin \theta &= n \cos p_0 - m' \sin p_0 \\ R' \sin \theta' &= -m \sin p_0 - n' \cos p_0 \\ R' \cos \theta' &= n \sin p_0 + m' \cos p_0 \\ P &= p \cos p_0 - p' \sin p_0 \\ P' &= p \sin p_0 + p' \cos p_0\end{aligned}$$

the quantities R , R' , θ , θ' , P , P' can be tabulated as a function of the absolute time, and the quantities Δ_1 and Δ_2 can be obtained in the form

$$\left. \begin{aligned}\Delta_1 &= k R \sin (H + \theta) + h P \\ \Delta_2 &= k R' \cos (H + \theta') + h P\end{aligned} \right\} \dots \dots \dots (47)$$

The computation of the quantities R , R' , etc., can be still farther simplified by computing the auxiliaries μ , μ' , μ'' , φ , φ' , and φ'' from the formulæ

$$\begin{array}{lll} \mu \cos \varphi = m & \mu' \cos \varphi' = n & \mu'' \cos \varphi'' = p \\ \mu \sin \varphi = n' & \mu' \sin \varphi' = m' & \mu'' \sin \varphi'' = p'\end{array}$$

when we shall have

$$\left. \begin{aligned}R \sin \theta &= \mu' \cos (\varphi' + p_0) \\ R \cos \theta &= \mu \cos (\varphi + p_0) \\ R' \sin \theta' &= -\mu \sin (\varphi + p_0) \\ R' \cos \theta' &= \mu' \sin (\varphi' + p_0) \\ P &= \mu'' \cos (\varphi'' + p_0) \\ P' &= \mu'' \sin (\varphi'' + p_0)\end{aligned} \right\} \dots \dots \dots (48)$$

The numerical values of these quantities for the case of the Transit of Venus are given subsequently.

§ 6. RECAPITULATION OF THE PRECEDING FORMULÆ FOR THE REDUCTION OF THE
TRANSIT OF VENUS PHOTOGRAPHS

For convenience in applying the preceding theory, we collect the formulæ deduced in the preceding pages, so far as they are necessary for the actual computation. The process of reduction and comparison is divided into three problems.

PROBLEM I.

From the measures on the photographic plate and the known constants pertaining to the photographic telescope, to find the apparent position angle and distance of the center of Venus referred to the center of the Sun.

The data required for solution are :

(1). The west hour angle and declination of that point of the celestial sphere toward which the rays emanating from the apparent center of the Sun were thrown by reflection from the heliostat, in order to be photographed (to $5''$).

(2). The apparent hour angle and declination of the center of the Sun, as affected by parallax and refraction (to $5''$). This is to be computed from the sidereal time.

(3). The distance of the center of Venus from that of the mean center of the Sun's disk, as measured on the photograph.

(4). The angle which the line joining the centers of Venus and the sun makes with the vertical at the Sun's center. This angle is to be counted from the line drawn from the center of the Sun's image, vertically upward in the northern hemisphere and downward in the southern hemisphere (toward the north in each case), in the direction E N, or the opposite of that in which the hands of a watch move.

(5). The reduced distance between the image on the sensitive plate and the second principal point of the photographic objective.

1. To express the solution we put—

a , the west azimuth of the central line of the photographic telescope as found from transit instrument;

b , the error of level of the middle horizontal line of the photographic plate, counted from the true horizon toward the north, so that a positive b means photographic plate too high in the northern hemisphere and too low in the southern;

$a' b'$, the corresponding azimuth and level of the center of the Sun's image relative to the cardinal lines on the plate, reduced to seconds of arc. Then

$a'' = a + a'$ will be the west azimuth of the reflected image of the sun's center.

Put, also,

δ_0 , the declination of that point of the horizon toward which the reflected rays are thrown, so that in the northern hemisphere $\delta_0 = \varphi - 90^\circ$, and in the southern $\delta_0 = 90^\circ - \text{south latitude}$. Then $\delta_2 = \delta_0 + b''$ will be the declination required in (1), and $H_2 = \alpha'' \sec \delta_0$ will be its west hour angle.

S. Ex. 31—8

2. Put

τ , the local sidereal time at which the photograph was taken.

τ_0 , the corresponding Greenwich sidereal time.

λ , the west longitude of the place from Greenwich, expressed in arc.

φ , its geographical latitude.

α , δ , the geocentric right ascension and declination of the Sun's center, to be taken from AIRY'S tables, with the argument τ_0 .

\mathcal{Z} , the sun's zenith distance (to be computed).

q , its parallactic angle.

Express τ in arc and compute

$$H = \tau - \alpha, \text{ the Sun's west hour angle.}$$

Then compute \mathcal{Z} and q by the following process: First find a quantity $\log k$ and an angle ψ by the formulæ

$$k \sin \psi = \sin \varphi$$

$$k \cos \psi = \cos \varphi \cos H$$

and then find $\sin \mathcal{Z}$, $\tan q$, $\cos \mathcal{Z}$, q and \mathcal{Z} , by the formulæ

$$\sin \mathcal{Z} \sin q = \cos \varphi \sin H$$

$$\sin \mathcal{Z} \cos q = k \sin (\psi - \delta)$$

$$\cos \mathcal{Z} = k \cos (\psi - \delta)$$

using six place logarithms in the computation.

Find the mean refraction and parallax corresponding to \mathcal{Z} from the table on page 53, and put

$$\Delta \mathcal{Z} = \rho + \pi = \mathcal{Z} - \mathcal{Z}''$$

the correction for parallax and refraction, π being the Sun's parallax in altitude and ρ its refraction. This quantity, $\Delta \mathcal{Z}$, is given in the fourth column of the table. Compute next

$$\left. \begin{aligned} \Delta H &= -\sin q \sec \delta \Delta \mathcal{Z} \\ \Delta \delta &= +\cos q \Delta \mathcal{Z} \\ \Delta q &= +\sin q \tan \delta \Delta \mathcal{Z} \end{aligned} \right\} \text{using four place logarithms.}$$

The quantities required will then be

$$H'' = H + \Delta H$$

$$\delta'' = \delta + \Delta \delta$$

$$q'' = q + \Delta q$$

3. Put

r , the distance of centers of images of Sun and Venus on the plate;

f , the reduced distance from the plate to the center of divergence of the photographic objective. Then

$$s = \frac{r}{f} \times 206265'' = [5.31443] \frac{r}{f}$$

will be the apparent angular distance of centers.

4. Put ω for the value of the position-angle described in (4), which is derived immediately from measures on the photographic plate. Compute A from the formula

$$\tan \frac{1}{2} A = \frac{\cos \frac{1}{2} (\delta'' - \delta_2)}{\sin \frac{1}{2} (\delta'' + \delta_2)} \cot \frac{1}{2} (H'' - H_2)$$

Then

$$p'' = A + \omega - a'' \tan \delta_0$$

will be the apparent position angle of Venus relative to the Sun's center.

PROBLEM II.

Having found p'' and s'' , or the apparent position angle and distance of Venus from the Sun's apparent center, to free them from the effect of atmospheric refraction.

The corrections to be applied for this purpose are two in number:

1. A small correction to reduce the center of the Sun's image, as measured on the plate, to the image of his true center.

2. Correction for the effect of refraction in changing the altitude of the center of the Sun and of the center of Venus.

1. Compute

$$V'' = p'' - q''$$

$$\delta'' \mathcal{Z} = 0.75 \rho n'_2 s^2$$

as a function of \mathcal{Z} , s being the Sun's semidiameter in arc, and n'_2 the quantity defined in the preceding section. The corrections in question will then be

$$\delta'' V = - \frac{\sin V''}{\sin s''} \delta'' \mathcal{Z}$$

$$\delta'' s = \cos V'' \delta'' \mathcal{Z}$$

The quantity $\delta'' \mathcal{Z}$ is tabulated on page 53, and it is indifferent whether we use V , V' , or V'' in the computation.

2. For the computation of the second and larger correction, it is necessary to have an approximate value of s and V as corrected. The theoretical values computed in Problem III will answer for this purpose. Then put

$$V = p - q$$

$$V' = \frac{1}{2} (V + V'')$$

$$s' = \frac{1}{2} (s + s'')$$

and compute $\log \rho$ from the refraction tables, having regard to the thermometer and barometer, and entering the tables with the argument

$$\mathcal{Z}'' = \mathcal{Z} - \Delta \mathcal{Z}.$$

The corrections of the second class for refraction will then be

$$\begin{aligned}\delta' s &= \rho \sin s' \{ \cot \mathcal{Z}' + (n'_1 - \cot \mathcal{Z}') \cos^2 V' \} \\ &\quad - \rho n'_2 \sin^2 s' \cos^3 V \\ \delta' V &= - (n'_1 - \cot \mathcal{Z}') \rho \sin V' \cos V' \\ &\quad + \rho n'_2 \sin s' \sin V' \cos^3 V' \\ \delta p &= \delta' V - \Delta q + \delta'' V \\ \delta s &= \delta' s + \delta'' s\end{aligned}$$

We shall then have, from the photographs,

$$\begin{aligned}s &= s'' + \delta s \\ p &= p'' + \delta p\end{aligned}$$

PROBLEM III.

To express the position angle and distance of Venus from the Sun's center in terms of the geocentric elements, and their corrections.

1. Compute, for the station, the quantities

$$\begin{aligned}h &= \rho \sin \varphi \\ k &= \rho \cos \varphi'\end{aligned}$$

Here ρ is the radius vector of the station and φ' the geocentric latitude.

2. Find Δ_1 and Δ_2 from the equations

$$\begin{aligned}\Delta_1 &= k R \sin (H + \theta) + h P \\ \Delta_2 &= k R' \cos (H + \theta') + h P'\end{aligned}$$

where the quantities R , R' , θ , θ' , P , and P' are to be taken from the table given subsequently with the argument $\tau_0 =$ Greenwich sidereal time. They are computed by means of the formulæ given in § 5.

3. Find s and Δp from

$$\begin{aligned}s \sin \Delta p &= \Delta_1 & p &= p_0 + \Delta p \\ s \cos \Delta p &= s_0 + \Delta_2 & s &= s_0 + \Delta s\end{aligned}$$

and the equation to be written will be

$$\begin{aligned}s &= s_0 + \Delta s + \frac{\Delta s}{8''.85} \delta \pi + \sin p \cos \delta_1 \delta A + \cos p \delta D \\ p &= p_0 + \Delta p + \frac{\Delta p}{8''.85} \delta \pi + \frac{\cos p \cos \delta_1}{\sin s} \delta A - \frac{\sin p}{\sin s} \delta D\end{aligned}$$

in which all the quantities except s , $\delta \pi$, δA , and δD are to be reduced to numbers.

§ 7. SPECIAL INVESTIGATION OF THE CONSTANTS PERTAINING TO EACH PHOTOHELIOGRAPH.

The most important of these constants is that which expresses the ratio between the measures on the collodion film and the angular distances of the points photographed. In order to determine this ratio, the reflecting surface should either be a perfect plane, or its curvature should be known with great accuracy. The strongest reason for this requirement is that a curved reflector will, in combination with the objective, form a compound system, the center of divergence of which will be displaced from that of the objective alone by a distance proportioned to the power of the mirror, relative to that of the objective, and to the distance of the mirror from the objective. It was found that, in order that the measures might be correct within their ten thousandth part, the radius of curvature of the reflector should not be less than four miles, and therefore its focal distance for parallel rays should not be less than two miles. More exactly, the differences among the curvatures of the reflectors should not exceed the limit here set. A common curvature of all the mirrors would, indeed, influence the measured distance of Venus from the Sun's center, but would affect the solar parallax deduced from the measures by only a small fraction of its entire amount. It was therefore important to compare the curvatures of the several mirrors. This was done in three ways:

1. A pair of 5'' achromatic telescopes, each of 71'' focal length, were focused upon each other, a straight edge being inserted in the focus of the one, and the other pointed into it; a similar edge was adjusted in the eye-piece of the second telescope in such a way as to correspond to the image of that in the first, and the two were accurately focussed on each other. The one telescope was then turned 90° around a point midway between the two objectives, and each of the mirrors was set up in succession with its center in this rotation point, and at an angle of 45° with each telescope, so that by looking into one telescope the image of the edge in the focus of the other could be seen by reflection. The amount by which the eye-piece had to be pushed in or out, to bring the one edge into focus with the reflected image of the other, was then measured. This showed the change in the focal length of the telescope produced by the curvature of the mirror. The following is the amount by which the focus appeared to be lengthened by each mirror:

	in.
Mirror No. 1	— 0.030
2	— 0.027
3	— 0.060
4	— 0.015
5	+ 0.010
6	— 0.020
7	— 0.025
8	— 0.025
	—
Mean	— 0.024

This mean corresponds to a radius of curvature of 35,000 feet, the mirrors being, on the whole, concave. This curvature, with the mirror a foot from the objective, would make the measures on the photographic plate erroneous by about $\frac{1}{17000}$ of their whole amount.

2. The relative curvatures of the mirrors could be compared with rather more precision by using them in succession in front of the photographic objective, and determining the distances between the positions in which the best images were formed. This plan was devised and put into execution by Dr. DRAPER. In order to get the best optical image with the photographic telescope, the rays were passed through a cell containing a solution of ammoniacal sulphate of copper just before reaching the focus. A frame, containing a piece of ground glass to receive the image, could be slid in and out near the focus, and its position on a ruled scale of inches measured at each setting. Care was taken that the observer who brought the frame into focus should be unconscious of its position, and that the scale should be read by another. The following table shows the readings taken by Dr. DRAPER and myself in this way. In column D are given DRAPER'S readings for focus, as taken on May 7, 1874, and in column N my own, each of which immediately preceded the corresponding one of his. In the following columns are given the separate results for the amount by which each focal length exceeded the mean; (1) and (2), from the measures just given; (3), from a set taken on the preceding day, and probably less accurate than those given; (4), from the measures with telescopes already given, and reduced to the photographic telescope by multiplying them by the square of the ratio of the focal lengths.

Mirror.	D. in.	N. in.	(1)	(2)	(3)	(4)	Mean.	Radius in feet.
1. Chatham Island .	14.7	13.8	+0.7	-0.1	-0.2	-0.3	0.0	∞
2. Queenstown . .	13.0	13.0	-1.0	-0.9	-0.1	-0.1	-0.5	+ 72.000
3. Peking	13.7	13.9	-0.3	0.0	-1.5	-1.5	-0.8	+ 45.000
4. Nagasaki . . .	14.6	13.9	+0.6	0.0	+0.4	+0.4	+0.4	- 90.000
5. Hobart Town .	14.5	14.7	+0.5	+0.8	+1.5	+1.4	+1.0	- 36.000
6. Campbelltown .	13.0	14.7	-1.0	+0.8	+0.2	+0.2	0.0	∞
7. Wladiwostok . .	14.2	12.8	+0.2	-1.1	0.0	0.0	-0.2	+ 180.000
8. Kerguelen . . .	13.8	14.6	-0.2	+0.7	0.0	0.0	+0.1	- 360.000
Mean	14.0	13.9						

Accepting these mean results, it would seem that, neglecting the curvature, the plate measures of the distance of centers of Venus and the Sun would be in error by amounts varying from zero to 0."05.

3. After the return of the instruments in 1875, the curvatures were compared by Mr. TODD with a very delicate spherometer from GRÜNOW, of New York, loaned to the Observatory by the U. S. Military Academy. The radius of the circle on which lay the three legs was only about 0^m.1 less than that of the mirror, and as it was hardly to be expected that the figure of the mirror would be perfect to its edge, considerable deviations were expected. The readings were made with the greatest care, the reflecting surfaces of the mirrors being made horizontal by a spirit-level before

the application of the instrument, and the screw being turned with the utmost delicacy until the three legs began to turn round the center. Mr. TODD gives the following statement of his trials:

In order to get a near approximation to the effect of supposed curvature in the entire mirror, the spherometer was applied to several parts of the surface, first by dividing one-third of the circumference of the mirror into three, and afterward into five, nearly equal arcs. A zero point of each mirror was therefore marked by a cross near the thinner-side of the mirror. The instrumental zero adopted for the spherometer was the pivot to which the upright scale of divisions is attached.

The value of one rotation of the screw of the spherometer is $0^{\text{in}}.01966^*$ The head of the screw carries a disk of about seven inches diameter, the circumference of which is graduated into five hundred parts. The radius of the circle passing through the pivots of the feet of the spherometer (that is, the distance from the axis of the screw to any one of the three pivots) is $3^{\text{in}}.543$.

In making the first reading for each mirror, the zero of the spherometer was placed coincident with the zero of the mirror, care being taken that the three feet of the instrument were at nearly equal distances from the nearest points of the circumference of the mirror. In this position the reading on the line " 0° " was made. Then, having moved the feet of the instrument through an arc of forty degrees, the reading " $0^{\circ} + 40^{\circ}$ " was made; and afterward, having moved forty degrees farther, the reading " $0^{\circ} + 80^{\circ}$ " was taken. The second scheme of readings gave five observations for each mirror—recorded in the lines " 0° ", " $0^{\circ} + 24^{\circ}$ ", " $0^{\circ} + 48^{\circ}$ ", etc. The first three decimals are in units of the divided head of the spherometer; the fourth decimal is the number of estimated tenths of the separate divisions, so that each unit of the fourth place measures nearly $0^{\text{in}}.000004$.

Some of the observations are here given. The letters in the headings of the several columns are the initial letters of the stations at which the mirrors were used. The Roman numerals underneath are the numbers of the several mirrors (see page 22).

The Kerguelen mirror (No. VIII) had not been returned from the station.

1876, *May 1, 1.30 p. m.* *Temperature, 59°.*

Position of Instr. on Mirror.	W. VII.	N. IV.	P. III.	K. VIII.	H. V.	C. VI.	Q. II.	Ch. I.
0°	0.1330	0.1334	0.1333	0.1336	0.1333	0.1334	0.1335
$0^{\circ} + 40^{\circ}$	0.1333	0.1332	0.1333	0.1334	0.1334	0.1334	0.1333
$0^{\circ} + 80^{\circ}$	0.1333	0.1333	0.1333	0.1336	0.1333	0.1332	0.1334
Means..	0.1332	0.1333	0.1333	0.1335	0.1333	0.1333	0.1334

The Queenstown mirror (No. II) was removed after these measures.

* See page 28 of *Reports on Telescopic Observations of the Transit of Mercury*, May 5-6, 1878, forming Appendix II of the *Washington Observations for 1876*.

TRANSIT OF VENUS, 1874.

1876, May 1, 2.45 p. m. Temperature, 59°.0.

Position of Instr. on Mirror.	W. VII.	N. IV.	P. III.	K. VIII.	H. V.	C. VI.	Q. II.	Ch. I.
0°	o. 1337	o. 1335	o. 1332	o. 1337	o. 1335	o. 1336
0° + 40°	o. 1336	o. 1334	o. 1332	o. 1335	o. 1335	o. 1335
0° + 80°	o. 1334	o. 1332	o. 1333	o. 1334	o. 1333	o. 1336
Means..	o. 1336	o. 1334	o. 1332	o. 1335	o. 1334	o. 1336

1876, May 1, 3 p. m. Temperature, 59°.0.

Position of Instr. on Mirror.	W. VII.	N. IV.	P. III.	K. VIII.	H. V.	C. VI.	Q. II.	Ch. I.
0°	o. 1334	o. 1335	o. 1334	o. 1334	o. 1334	o. 1335
0° + 40°	o. 1335	o. 1334	o. 1333	o. 1334	o. 1334	o. 1335
0° + 80°	o. 1335	o. 1333	o. 1332	o. 1334	o. 1333	o. 1334
Means..	o. 1335	o. 1334	o. 1333	o. 1334	o. 1334	o. 1335

1876, May 1, 3.20 p. m. Temperature, 59°.5.

Position of Instr. on Mirror.	W. VII.	N. IV.	P. III.	K. VIII.	H. V.	C. VI.	Q. II.	Ch. I.
0°	o. 1334	o. 1334	o. 1333	o. 1336	o. 1334	o. 1335
0° + 24°	o. 1335	o. 1335	o. 1333	o. 1335	o. 1336	o. 1336
0° + 48°	o. 1335	o. 1334	o. 1333	o. 1335	o. 1334	o. 1336
0° + 72°	o. 1335	o. 1334	o. 1335	o. 1334	o. 1336	o. 1336
0° + 96°	o. 1334	o. 1333	o. 1333	o. 1336	o. 1335	o. 1335
Means..	o. 1335	o. 1334	o. 1333	o. 1335	o. 1335	o. 1336

1876, May 2, 1.10 p. m. Temperature, 59°.5.

Position of Instr. on Mirror.	W. VII.	N. IV.	P. III.	K. VIII.	H. V.	C. VI.	Q. II.	Ch. I.
0°	o. 1333	o. 1335	o. 1332	o. 1334	o. 1334	o. 1333
0° + 24°	o. 1333	o. 1334	o. 1332	o. 1334	o. 1333	o. 1333
0° + 48°	o. 1333	o. 1334	o. 1333	o. 1333	o. 1334	o. 1332
0° + 72°	o. 1333	o. 1333	o. 1333	o. 1334	o. 1335	o. 1333
0° + 96°	o. 1333	o. 1332	o. 1333	o. 1335	o. 1334	o. 1333
Means..	o. 1333	o. 1333	o. 1333	o. 1334	o. 1334	o. 1333

1876, May 2, 2.15 p. m. Temperature, 64°.0.

Position of Instr. on Mirror.	W. VII.	N. IV.	P. III.	K. VIII.	H. V.	C. VI.	Q. II.	Ch. I.
0°	o. 1333	o. 1334	o. 1334	o. 1336	o. 1335	o. 1334
0° + 24°	o. 1333	o. 1333	o. 1333	o. 1334	o. 1334	o. 1333
0° + 48°	o. 1334	o. 1333	o. 1333	o. 1334	o. 1334	o. 1333
0° + 72°	o. 1334	o. 1334	o. 1334	o. 1334	o. 1335	o. 1334
0° + 96°	o. 1333	o. 1333	o. 1333	o. 1334	o. 1335	o. 1334
Means..	o. 1333	o. 1333	o. 1333	o. 1334	o. 1335	o. 1334

In making the observations on May 2, the legs of the spherometer were protected from change of temperature in handling by a covering of chamois skin.

The spherometer measures would show that there is absolutely no difference among the mirrors exceeding four millionths of an inch, a depression which, if arising from regular curvature, would correspond to a radius of curvature of 15 miles. We have therefore a slight discordance between the optical tests applied by Dr. DRAPER and myself and these spherometer tests by Mr. TODD. The latter are certainly the more free from accidental errors, and I should accept their results without hesitation but for two reasons. One is the possibility of a slight sticking of the screw at the particular point corresponding to perfect flatness of the support, whereby it might tend to show the same reading through the space of $\frac{1}{100000}$ of an inch change in the support; the other, that the form of the mirrors when in actual use, exposed to the Sun's rays, may be a little different from that which they assume when laid flat. If we determine the curvatures entirely from the optical measures, we must correct the column "Mean" in the preceding table by -1.0 for mean concavity of the eight mirrors determined by the telescopic measures. We shall thus have the amounts in the next table by which the astronomical focus is shortened by the concavity of the mirrors. By this concavity the image will be enlarged in the ratio

$$1 + \frac{2D}{\rho}$$

ρ being the radius of curvature, and D the distance of the mirror from the objective. If we put s for the amount by which the focus is shortened, we shall have with sufficient approximation

$$\rho \text{ (in feet)} = \frac{36000}{s \text{ (in inches)}}$$

We may suppose the value of D to be one foot, and shall then have

$$\frac{2D}{\rho} = \frac{s}{18000}$$

The table shows the values of s , and of the resulting factors by which the distance from the center of divergence to the ruled plate must be corrected to reduce to what it would have been had the mirrors been perfectly flat.

	s	$F=1 +$	$\log F$
Wladiwostok	+ 1.0	+ .00006	+ .000024
Nagasaki	+ 1.5	+ .00008	+ .000036
Peking	+ 1.8	+ .00010	+ .000043
Kerguelen	+ 0.6	+ .00003	+ .000014
Hobart Town	0.0	.00000	.000000
Campbelltown	+ 1.0	+ .00006	+ .000024
Queenstown	+ 1.2	+ .00007	+ .000029
Chatham Island	+ 0.9	+ .00005	+ .000022

Centers of divergence of the objectives.

In order to compute the positions of these centers, it is necessary to know the form, thickness, and index of refraction of each of the lenses composing the objectives. The thicknesses were measured near the edges with a calliper reading to .001 of an inch. The entire objectives were thicker near the center by .0036 of an inch, a difference of which it was not deemed necessary to take account, in view of the fact that rays near the edge have most influence in forming the image.

The curvatures were first determined with a small spherometer, the feet of which were on a circle of 1^m.25 radius. It was found that, within so small a circle, the deviation was so small that the instrument did not certainly indicate any differences among the several glasses, though it was known that differences must exist, since all were not of absolutely the same focal length. Recourse was therefore had to optical methods. The concave surfaces were investigated by setting the flint lenses in a vertical position, and finding the position of the focus for reflected rays. The process is too simple to require a detailed description.

To investigate the several convex surfaces, a 5" telescope, of six feet focal length, was set up horizontally and its stellar focus found by setting up one of the plane mirrors immediately in front of the objective, and "taking the shade" on a lamp placed a little inside the focus*. The mirror was then removed and each convex surface of the eight crown glasses was, in succession, placed in front of the objective, and the focal point of the rays reflected from each was determined. As the great radius of curvature of the first face of the flint glasses made their direct determinations troublesome, these faces were treated in the same manner, but, being slightly concave, were also determined by finding their center of curvature by reflection from a distance equal to the radius of each.

If f_1 and f_2 be the respective distances of a pair of conjugate principal foci from their corresponding centers of convergence, the astronomical focal distance f is well known to be given by the equation

$$f = \frac{f_1 f_2}{f_1 + f_2}$$

The positions of the centers of convergence of the objective used in the determinations were not computed, as the error which would arise from supposing them both to be situated in the outer surface of the objective could not have any serious influence on the present determinations of the curvatures of the photographic objectives. A rough

* The process of making the measures was as follows: A graduated bar was placed nearly parallel with the axis of the five-inch telescope, one end of the bar being in contact with the eye-piece end of the telescope tube. In "taking the shade", a lamp shining through a slit was placed an inch or two inside the focus, so that the image of the slit, formed by reflection from the glass surface outside the objective, fell a little outside the lamp. A small try-square was set up vertically upon the graduated bar, near the conjugate focus, and by moving it backward and forward in a direction perpendicular to the axis of the telescope, the pencil of light emanating by reflection from the surface of the photographic lens could be intercepted before reaching the eye of the observer. If, in moving the try-square, the surface of the photographic lens appeared to lose its illumination in the same direction as the square was moved, the "shade", or try-square, was too near the reflector; if in the opposite direction, the shade was too far from the reflector, or beyond (that is, outside of) the conjugate focus. By a tentative process, then, that point was readily found where, moving the square very minutely across the axis of the pencil, the illumination of the entire surface disappeared simultaneously. This point marked the conjugate focus. The operation was several times repeated for all the convex surfaces of the photographic lenses, and the mean readings from the graduated horizontal bar were considered as defining the position of the conjugate foci.

In investigating the short radius concave surfaces of the flint lenses, the shade was taken on the direct reflection in the same manner, but entirely without the intervention of the achromatic objective of the horizontal telescope.

approximation would be attained by supposing the first one to be .05 of an inch outside the outer face of the objective, and the second one to be 0^m.25 within it.

By reflection from the plane mirror, the distances of conjugate foci, both on the flint side of the objective, were found to be

$$\begin{aligned} f'_1 &= 5.951 \text{ ft.} \\ f'_2 &= 6.173 \end{aligned}$$

The accents indicate that the distances are measured from the outer crown-face of the objective, and not from the center of convergence. We therefore have for the position of the astronomical focus

$$f'' = 6.060$$

As a check upon this determination, the positions of three pairs of conjugate foci were found by placing the lamp about 16 feet in front of the objective, and finding the points of the corresponding focus beyond the eye-end. Thus was found

$$\begin{aligned} \text{Values of } f''_1, & 15.957, 16.23, 15.78 \\ \text{Corresponding values of } f''_2, & 9.744, 9.614, 9.789 \end{aligned}$$

Supposing $f_1 = f'_1 - .004$, $f_2 = f'_2 - .021$, we have, from the formula,

$$\begin{aligned} \text{Corresponding values of } f, & 6.041, 6.030, 6.034 \\ \text{Corresponding values of } f', & 6.062, 6.051, 6.055 \end{aligned}$$

The mean result is .004 less than that found by reflection from the mirror. The former has been used for the slightly concave faces of the flint, but the latter, being determined by placing the lamp at a distance very nearly equal to the radius of the convex surfaces of the crowns, was used in determining the curvature of those surfaces. The following table shows, for various focal distances behind the objective, the respective distances of the corresponding conjugate foci in front of the objective, and hence the distance of the center of curvature of the convex lens tried from the front face of the objective. The distance between this face of the objective and the reflecting face of the lens to be tried being .020 of a foot, the actual radius of curvature of the reflecting surface of the lens will be .020 foot less.

Table of conjugate foci, etc.

f'_2 (feet.)	f'_1 (feet.)	Radius of reflecting surface. (feet.)
9.63	16.224	16.204
9.64	16.195	16.175
9.65	16.167	16.147
9.66	16.139	16.119
9.67	16.111	16.091
9.68	16.084	16.064
9.69	16.057	16.037
9.70	16.030	16.010
9.71	16.003	15.983
9.72	15.976	15.956
9.73	15.950	15.930

The convex surfaces were now investigated, as follows :

The lamp was placed behind the objective, so that the slit through which it shone was 9.544 feet from the front (crown) surface. The sixteen surfaces of the eight lenses to be examined were then successively placed in front of the objective, at a distance of .020 of a foot, and the conjugate focus was found for the rays from the lamp, which, after passing through the objective and being reflected from the surface of the lens, were returned through the objective and brought to a focus as near the lamp as practicable. The data and results are given in the following table, which gives

- f'_3 , the distance of the slit of the lamp from the front face of the objective ;
 f'_4 , the observed distance of the corresponding focus, formed by reflection, as just described ;
 f'_2 , the distance at which the lamp should have been placed in order that the slit and its image should coincide ; that is, in order that all the rays, after passing through the objective, should have been normal to the reflecting surface of the lens, and therefore should have met at the center of curvature of the surface ;
 r , the corresponding radius of curvature of the surface, r_1 referring to the outer surface and r_2 to the inner one. r is taken immediately from the above table. The value of f'_2 is found by the formula

$$f'_2 = \frac{f'_3 f'_4}{f'_3 + f'_4}$$

	f'_3	First face of Crown.			Second face of Crown.			Second face of Flint.			
		f'_4	f'_2	r_1	f'_4	f'_2	r_2	f'_3	f'_4	f'_2	r_4
Wladiwostok	Feet. 9.544	Feet. 9.792	Feet. 9.666	Feet. 16.102	Feet. 9.837	Feet. 9.688	Feet. 16.044	Feet. 5.844	Feet. 6.009	Feet. 5.925	Feet. 266
Nagasaki	"	9.854	9.696	16.021	9.874	9.706	15.994	"	5.970	5.906	234
Peking	"	9.810	9.675	16.078	9.856	9.697	16.018	"	5.932	5.887	208
Kerguelen	"	9.902	9.720	15.956	9.819	9.679	16.067	"	5.974	5.908	236
Hobart Town	"	9.726	9.634	16.192	9.824	9.682	16.059	"	5.998	5.920	256
Campbelltown	"	9.808	9.674	16.080	9.842	9.691	16.034	"	5.980	5.911	240
Queenstown	"	9.880	9.709	15.986	9.800	9.670	16.091	"	5.980	5.911	240
Chatham Island	"	9.836	9.688	16.042	9.848	9.693	16.029	"	6.008	5.925	266

As already stated, the curvatures of the first surfaces of the flint lenses were determined by reflection, without the intervention of an objective. The results are given in the following table, together with the thicknesses of the lenses as measured. Here the measures are all reduced to English inches, and algebraic signs are given to the radii of curvature, to accord with the theory of the subject ; that is, a surface of positive curvature is one in which the center is on the side from which the ray comes, and one of negative curvature on the side toward which it goes.

	Thickness.		Radii of curvature.			
	Crown.	Flint.	r_1	r_2	r_3	r_4
	Inches.	Inches.	Inches.	Inches.	Inches.	Inches.
Wladiwostok	0.410	0.530	— 193.22	+ 192.53	+ 189.88	— 3192
Nagasaki	0.434	0.488	— 192.25	191.93	190.30	— 2808
Peking	0.426	0.481	— 192.94	192.22	189.82	— 2496
Kerguelen	0.283	0.425	— 191.47	192.80	190.51	— 2832
Hobart Town	0.440	0.443	— 194.30	192.71	189.54	— 3072
Campbelltown	0.389	0.499	— 192.96	192.41	191.27	— 2880
Queenstown	0.466	0.463	— 191.83	193.09	190.42	— 2880
Chatham Island	0.391	0.454	— 192.50	+ 192.35	+ 189.70	— 3192

The specific gravities of the lenses were determined at the Coast Survey Office for the objectives of longest and shortest focus, and found to be sensibly the same. The mean result was as follows :

Specific gravity of crown, 2.5622
Specific gravity of flint, 3.2176

The specific gravities do not directly enter into the determinations which we seek, but only the indexes of refraction. The accurate determination of these indexes, though not impracticable, would be troublesome, and it was judged that values near enough for the present purpose could be inferred from the specific gravities. From the equalities of the specific gravities in the case of the two extreme glasses weighed, it was concluded that all the lenses were made from the same pots; this, however, is not certain.

The region of greatest photographic intensity in the spectrum being situated between G and H, it was deemed sufficient to seek the indexes of refraction for these rays. Dr. J. HOPKINSON gives, in the Proceedings of the Royal Society for 1877, a list of indexes of refraction for glasses of various specific gravities, from which the following results are extracted :

Hard crown,	sp. gr. = 2.48575;	(n G) = 1.52835;	(n H) = 1.53279
“	“ 2.48664	“ 1.52886	“ 1.53332
Soft crown,	“ 2.55035	“ 1.52660	“ 1.53142
Extra light flint,	“ 2.86636	“ 1.55638	“ 1.56276
Light flint,	“ 3.20609	“ 1.59282	“ 1.60072
Dense flint,	“ 3.65865	“ 1.64607	“ 1.65622
Extra dense flint,	“ 3.88947	“ 1.67702	“ 1.68857
Double extra flint,	“ 4.42162	“ 1.74321	“ 1.75778

From the specific gravities of the photographic objectives, it seems that they belong to the classes soft crown and light flint. The crowns are a little more dense than Dr. HOPKINSON'S specimen, but I cannot say whether this increase of density would result in an increase or diminution of the index of refraction. An increase would be es-

teemed the natural result, but it seems that the soft crown, though more dense than the hard, has less refractive power. The effect being doubtful, we may suppose the indexes the same as in Dr. HOPKINSON'S specimen. The indexes for the flint may be readily interpolated from the table. We thus have, for the most probable values of the indexes of the photographic objectives:

Crowns: Density, 2.5622; (n G) = 1.5266; (n H) = 1.5314; $\mathcal{A} = 48$
 Flints: Density, 3.2176; (n' G) = 1.5941; (n' H) = 1.6021; $\mathcal{A} = 80$

The corresponding dispersive powers for the region from G to H are in the ratio of 48:80 or 3:5, while the corresponding powers of the lenses, as dependent on their curvatures alone, are in the ratio of about 15:8. This would indicate a decided under-correction, even for the photographic rays, in this region of the spectrum.

We have now to compute the focal length and the second center of divergence from the curvatures of the lenses and the indexes of refraction. This may be done by the following formulæ. Put,

r_1, r_2, r_3, r_4 , the radii of curvature of the four refracting surfaces, in the order in which the light passes them;
 n , the index of refraction for the crown lenses;
 n' , the same, for the flint lens;
 t_1, t_2, t_3 , the thicknesses of the spaces between the refracting surfaces, divided by the respective indexes of refraction, the first and third referring to the crown and flint glasses, respectively; the second, to the air-space between them. As nearly as could be determined, the thickness of the air-space was about .004 of an inch in all the glasses.

We then compute

$$u = \frac{n-1}{r_1} \quad u' = \frac{1-n}{r_2}$$

$$u'' = \frac{n'-1}{r_3} \quad u''' = \frac{1-n'}{r_4}$$

These four values of u may be considered as representing the *powers* of the four surfaces.

$$h = t_1 + t_2 + t_3$$

$$k = u + u' + u'' + u'''$$

$$l = 1 + u' t_1 + u'' (t_1 + t_2) + u''' (t_1 + t_2 + t_3)^*$$

$$g = 1 + u (t_1 + t_2 + t_3) + u' (t_2 + t_3) + u'' t_3$$

Then

$$f' = -\frac{g}{k}$$

will be the distance of the focus from the fourth surface, or inner face of the flint lens, and

$$s = \frac{g-1}{k}$$

will be the distance of the center of divergence inside the same face.

* The value of l is not wanted in the case of photographs taken in the astronomical focus.

It is not to be expected that the value of f' will agree exactly with the measured focus, owing to the uncertainty of the values of the measured curvatures and assumed indexes of refraction. The value of k derived from the equation

$$k = -\frac{g}{f'}$$

f' being the focal distance found by actual trial, will probably be more accurate than that found by computation. If we substitute this in the equation for s , we shall have

$$s = \frac{f'(1-g)}{g}$$

Practically, it is indifferent which value of k is used.

From the curvatures already given, we find the following values of the quantities sought for the two assumed indexes:

$$\text{Data } \left\{ \begin{array}{l} \text{I. Ray G, } n = 1.5266; n' = 1.5941 \\ \text{II. Ray H, } n = 1.5314; n' = 1.6021 \end{array} \right.$$

Station.	h	k		g		s	
	G and H.	Ray G.	Ray H.	Ray G.	Ray H.	Ray G.	Ray H.
Wladiwostok	0.603	— 0.002145	— 0.002150	0.99848	0.99847	0.709	0.713
Nagasaki593	.002150	.002155	.99848	.99847	.707	.709
Peking583	.002101	.002105	.99852	.99851	.707	.708
Kerguelen455	.002152	.002158	.99884	.99884	.538	.539
Hobart Town568	.002114	.002119	.99856	.99855	.682	.682
Campbelltown570	.002153	.002159	.99855	.99854	.674	.675
Queenstown598	.002146	.002151	.99846	.99845	.717	.719
Chatham Island544	— 0.002156	— 0.002160	0.99861	0.99861	0.643	.644

The computed focal lengths are as follows. We give, for comparison with them, the actual distances from the inner face of the objective to the collodion film as the instruments were adjusted by the observers. The lengths are in English inches:

	f' computed.		f' , as adjusted at the station.
	Ray G.	Ray H.	
Wladiwostok	465.5	464.4	462.5
Nagasaki	464.4	463.3	461.2
Peking	475.3	474.4	472.8
Kerguelen	464.1	462.9	462.7
Hobart Town	472.4	471.2	465.1
Campbelltown	463.8	462.5	462.2
Queenstown	465.3	464.2	463.2
Chatham Island	463.2	462.3	461.1

In the case of Hobart Town the discrepancy is so great as to suggest the probability that the glass of one of the lenses was different from that used in the corresponding lenses of the other objectives. In all the other objectives the agreement is as good as could be expected, considering the uncertainty of the indexes of refraction,

and the difficulty of finding the best focus within the range of an inch, by trial. The important quantities which are the object of this investigation are the distances of the centers of divergence from the inner face of the objective, and these seem to be determined with all the precision that can be desired.

Distances between the inner or last face of the objective, and the outer or first face of the ruled plate in the plate-holder.

The method of measuring these distances has already been described, and it now remains to collect and discuss the results. The distance, as measured, is made up of three parts:

1. The distance between the face of the objective and the silver plumb-line suspended from the end of the long rod, as measured with the jaw micrometer.
2. The length of the rod itself.
3. The distance between the plumb-line at the other end of the rod and the outer face of the ruled plate.

The lengths of the several rods were determined at the Coast Survey Office before the departure of the expeditions, and at the Observatory after their return. The last determinations were made by comparison with standard ten-foot and five-foot steel rods of the Coast Survey Office, loaned for the purpose. To make the measures, a level platform 40 feet long was constructed in the transit-circle room of the Observatory. A steel straight-edge was firmly fastened across one end of this platform, at a height of about one-fourth of an inch, and the end of each rod to be measured was brought into contact with this edge. The measures were made by placing the steel rods alternately end to end along the iron rod to be measured, commencing at the straight edge. The excess of the rods over 35 feet was measured with DARLING, BROWN & SHARP'S standard scales, allowance being made for the fact that each of them was ^m.006 longer than the Coast-Survey standard. As a check upon all the measurements, the several rods were also compared differentially, so that there were two results for each rod, one an absolute measure, and the other the result of a comparison with the absolute measures of the others. As no doubt or difficulty was experienced in making the determinations with all necessary certainty, it is deemed unnecessary to enter into details, or give more than the final results for length of the rods. The following table shows the lengths, first as given by Mr. JOHN CLARK, of the Coast Survey, before the departure of the expeditions, and then as finally determined by the method just described:

Station.	No. of rod.	Length. (U. S. C. S.) Inches.	Length. (Observatory.) Inches.	Length. (To notch.)
Wladiwostok	IV	450.34	450.357
Nagasaki	II	450.42	450.437
Peking	I	461.42	461.425
Kerguelen	III	453.50	453.488	451.514
Hobart Town	VII	453.50	453.498
Campbelltown	VI	451.96	451.946	449.717
Queenstown	VIII	451.50	451.491
Chatham Island	V	449.49	449.485

It may be assumed that these lengths correspond to a temperature of 62° Fahr.

The Kerguelen and Campbelltown rods were a little too long for the measurement of the focal lengths of the objectives; a narrow groove in which to hang the plumb-line was therefore cut around each of them at the following distances from the end of the rod :

Kerguelen, 1.974
Campbelltown, 2.229

These quantities are therefore to be subtracted from the lengths of the rods.

The next step is to determine the virtual thickness of the reticule plate, as affecting the size of the image ; that is, its actual thickness divided by its index of refraction for the photographic rays. The measured thicknesses, as determined by Professor HARKNESS, the indexes of refraction, and the virtual thicknesses thence concluded, are shown in the following table :

Station.	Thickness of Plate.	Index of Refraction.	Virtual Thickness.
Wladiwostok	0.327	1.540	0.212
Nagasaki.....	0.312	1.540	0.203
Peking.....	0.301	1.555	0.194
Kerguelen	0.339	1.548	0.219
Hobart Town.....	0.320	1.555	0.206
Campbelltown	0.329	1.555	0.211
Queenstown.....	0.344	1.540	0.223
Chatham Island.....	0.322	1.540	0.209

The remaining element required for the distance between the center of divergence of the objective and the sensitive collodion film is the distance between this film and the ruled surface of the reticule plate. A correction for the possible contraction or expansion of the collodion film arising from change of temperature or other causes will also be necessary. But it has been deemed best to avoid corrections of this sort *by referring all the measures to the ruled surface of the reticule plate as the fiducial focus*. The ruled lines on this plate being photographed with the sun on every image of the latter, it follows that, by comparing any distance on the photographic plate with the corresponding one on the ruled plate, we shall have the excess of the image on the collodion film over that on the reticule plate, as due to all causes whatever. The ratio of any pair of such distances will be the factor by which distances on the reticule are increased on the collodion film. Then dividing the measures on the collodion film by this factor, we shall reduce them to what they would have been had they been impressed upon the ruled surface of the reticule plate itself. The actual measures for this purpose will be given subsequently. From them it is concluded that there is no evidence of any unequal expansion or contraction of the collodion film depending on any cause whatever, but that the measures on the collodion may be reduced to measures upon the ruled plate by being multiplied by a factor which is constant for each station. The values of

this factor, which is a little less than unity, as found by actual measurement, are shown in the following table in the column F :

	F=1—	D inch.	$\frac{D}{f}$	Diff.	log F
Wladiwostok00038	0.171	.00037	— .00001	— .000165
Nagasaki00040	.178	.00039	— .00001	— .000174
Peking00038	.149	.00031	— .00007	— .000165
Kerguelen00033	.169	.00036	+ .00003	— .000143
Hobart Town00032	.155	.00033	+ .00001	— .000139
Campbelltown00039	.169	.00036	— .00003	— .000169
Queenstown00031	.145	.00031	— 00000	— .000135
Chatham Island00037	.140	.00030	— .00007	— .000161

In order to compare these results with those which would have been attained had the measures been continued to the collodion film, and had no allowance been made for changes in the film, due to differences of temperature or other causes, we present in column D the measured distances between the reticule plate and the film. In the case of Peking, and a few others, this distance is doubtful by one or two hundredths of an inch, as the parties were not supplied with any special apparatus for determining it, and the reticules were, through inadvertence, removed from the plates after the return of the latter, before the measurements were made at Washington. Dividing these numbers by f , the focal distance, we have the ratio which the excess of the images on the collodion film over those on the reticule plate should bear to the entire distance measured. In the last column are given the differences between these ratios and the measured values of $1 - F$. These differences probably arise from changes in the collodion film, due partly to the different temperatures to which it was subjected in the process of exposure and development. There can, I conceive, be no reasonable doubt that the factors F are the proper ones to pass from measures on the collodion film to those on the ruled scale.

Another reduction is that for difference of temperature of the reticule plate at the time of taking the photograph, and at the time of measuring the photograph. The measures being all made with a glass scale, of which the co-efficient of expansion may be assumed to be the same as in the reticule plate itself, this reduction will be most simply effected by taking for the length of a scale-division the value which corresponds to the temperature at which the photograph was taken. These temperatures may be assumed constant for the whole transit at each station except Peking. The following are the average values of this element in the photographic rooms, which we take for the temperatures of the reticules :

Wladiwostok,	temp. = 67° F.
Nagasaki,	“ 64
Peking,	“ 57 to 77
Kerguelen,	“ 59
Hobart Town,	“ 80
Campbelltown,	“ 79
Queenstown,	“ 73
Chatham Island,	“ 73

The value of the scale-division, as determined by Professor HARKNESS from a Coast Survey scale, is, in fractions of an inch,

$$\begin{aligned} & .019\ 886\ 9 + .000\ 000\ 096\ 4 (\tau - 32^\circ) \\ \text{or} & .019\ 889\ 8 + .000\ 000\ 096\ 4 (\tau - 62^\circ) \end{aligned}$$

As some doubt must exist whether this scale was strictly comparable with the steel rod which was used in determining the lengths of the station rods, it was deemed advisable to check it by such comparison as would involve no hypothesis respecting the comparability of different scales. This was effected by the intervention of a pair of DARLING, BROWN & SHARP two-foot steel scales. By using these as end measures for determining the lengths of the station rods, it was found that these lengths came out $0^{\text{in}}.12$ shorter than by the Coast Survey steel rods. It was thence concluded that these scales, used in this way, exceed the Coast Survey standard two feet by $0^{\text{in}}.006$. Professor HARKNESS then determined the combined length of the two scales, used as end measures, in terms of the glass scale of his measuring engine. This was done by starting from an arbitrary division on one scale and measuring consecutive steps of 5 inches. When the end of the scale was reached, the end of the other scale was brought into contact with it, and the measurement continued along that other scale. When the end of this scale was reached, that of the first scale was brought into contact with it, and the measurement continued until the starting point was reached. The total length measured was then the combined lengths of the two rods, independent of all errors of the scales. Thus was found

$$48.012 \text{ Coast Survey inches} = 2413.76 \text{ divisions of engine at } 62^\circ$$

which gives

$$1 \text{ div.} = 0^{\text{in}}.0198910.$$

This is greater by $.0000012$, or $\frac{1}{17000}$ of its entire amount, than the value at the same temperature given by the first investigation. Notwithstanding that the last is the result of a direct comparison, I consider it entitled to no greater weight than the other, for the reason that the arrangements for measuring the station rod with the two-foot rules were not so perfect as to preclude the possibility of an error. I shall therefore take the mean of the two results, putting

$$1 \text{ div.} = 0^{\text{in}}.019\ 890\ 4 + .000\ 000\ 096\ 4 (\tau - 62^\circ)$$

We may remark that the difference of the two results corresponds to a difference of $0''.05$ in the least distance of centers of Venus and the Sun, but that the value of the solar parallax will be appreciably the same whichever length is used.

We have now all the data necessary for determining the value of one division of the measuring engine in seconds of arc. Firstly, we have the following values for the reduced distance between the center of divergence of the objective and the reticule plate on which the fiducial images were formed. This distance is made up of three parts:

(1) The distance from the center of divergence to the second face of the flint lens, which we have called s .

- (2) From the second face of the flint to the first face of the reticule plate. The measures of this distance will be given in Part II. The result we call Σ .
- (3) The reduced thickness of the reticule plate.

The mean results for these lengths are collected as follows :

	(1) s (inches.)	(2) Σ	(3) (Reticule.)	f (Inches.)	log f	Corr'n for concavity of mirror.	log f corrected.
Wladiwostok,	0.711	462.214	0.212	463.137	2.665709	+ 24	2.665733
Nagasaki,	0.708	461.008	0.203	461.919	2.664566	+ 36	2.664602
Peking,	0.708	472.624	0.194	473.526	2.675344	+ 43	2.675387
Kerguelen,	0.538	464.480	0.219	465.237	2.667674	+ 14	2.667688
Hobart Town,	0.682	464.782	0.206	465.670	2.668078	0	2.668078
Campbelltown,	0.674	461.972	0.211	462.857	2.665447	+ 24	2.665471
Queenstown,	0.718	463.015	0.223	463.956	2.666477	+ 29	2.666506
Chatham Island,	0.644	460.903	0.209	461.756	2.664413	+ 22	2.664435

Next we require the value of one division of the measuring engine, reduced to the same scale, at the temperature of the reticule during the taking of the photographs. In the following table we give—

- (1) The length of the division, as already given, reduced for temperature.
- (2) The logarithm of this length.
- (3) The logarithm of the factor for reducing measures on the collodion film to measures on the reticule.
- (4) The concluded logarithm of the value, in seconds of arc, of the unit of scale-measure on the photographic plate, found from the formulæ $S = 206265'' \frac{D}{f}$, D being the length of one division of the measuring engine.

	τ	(1) D	(2) log D	(3)	(4) log S
Wladiwostok	67	0.0198909	8.298654	— 165	0.947181
Nagasaki	64	.0198906	8.298648	— 174	.948296
Peking	57	.0198899	8.298633	— 165	.937506
“	77	.0198918	8.298674	— 165	.937549
Kerguelen	59	.0198901	8.298637	— 143	.945231
Hobart Town	80	.0198921	8.298681	— 139	.944889
Campbelltown	79	.0198920	8.298678	— 169	.947463
Queenstown	73	.0198915	8.298667	— 135	.946451
Chatham Island	73	.0198915	8.298667	— 161	0.948496

In the case of Peking the temperature in the photograph house varied so widely during the transit that it is deemed advisable to use two values of log S, one for the four photographs taken just after the beginning, the other for those near the end. The adopted values are

Photographs 15-22	log S = 0.937510
Photographs 44-72	0.937540

CONSTANTS FOR DETERMINING THE POSITION ANGLE.

From the theoretical discussion already given, it will be seen that the constants required for deducing the position angle relative to the true meridian from that relative to the image of the plumb-line on the photographic plate are the latitude of the place and the errors of level and azimuth of the photographic telescope. The mean results for these errors, as given by the determinations at the different stations, are here set forth. The separate observations on which they are founded will be given in the reports of the chiefs of parties.

Station.	Azimuth of Telescope, Plate Holder west.	Plate Holder High.	Latitude of Station.
	"	"	° ' "
Wladiwostok	- 16	0	+ 43 6 35.6
Nagasaki	+ 14	+ 4	+ 32 43 21.1
Peking	+ 6	- 9	+ 39 54 15
Kerguelen	+ 13	- 17	- 49 21 22.1
Hobart Town	+ 2	- 5	- 42 53 24.6
Campbelltown	+ 5	- 4	- 41 55 42.9
Queenstown	- 4	- 18	- 45 2 7
Chatham Island	+ 3	- 3	- 43 49 3.2

The error of level of the photographic telescope at Wladiwostok has to be regarded as zero, since it was not determined in the way prescribed. A description of the process actually employed is given by Professor HALL in his report. The method employed, though not satisfactory, does not appear likely to be affected with a probable error greater than that of the plate measures.

§ 8. REDUCTIONS OF THE PHOTOGRAPHS IN TABULAR FORM.

The principal steps in the reductions of the photographs by the preceding method are shown in the following tables. The data required are enumerated in § 6, pp. 57-59, under the head of Problem I. Data (1) and (2) are computed astronomically by well known methods, and need no further elucidations. Data (3) and (4) are derived from measures of the photographic plates, executed by Prof. WILLIAM HARKNESS, U. S. N., and reduced under his direction so as to give the required quantities s and ω . Datum (5) is deduced in § 7 in the discussion of the photographic objectives. The following is the explanation of such of the columns as seem to need it.

Greenwich sidereal time.—The chronometer time at which each photograph was taken, and the correction of the chronometer on local sidereal time, will be given in their appropriate tables in connection with the observations at each station. To this local time the provisional west longitude given in Chapter II, p. 21, is applied, with the symbolic correction $\delta \lambda_i$, the index i representing the several stations in order.

Distance of centers.—Here is given, firstly, the distance as measured by Professor HARKNESS by the methods to be described in connection with the measures, and,

secondly, this distance reduced to seconds of arc by the factors just given. The actual measures and their discussion are reserved for a subsequent part.

Correction for refraction.—These corrections are computed by the formulæ, and table on page 53.

Angle ω .—This, like the distance, is derived from the plate measures, being the angle of position of the centers of the images of Venus and the Sun relative to the vertical line photographed on the plate.

The position angle p'' is that derived from the angle ω , as measured on the photographic plate, by the formulæ of §§ 4 and 6. No theoretical quantities enter into it except those dependent on the Sun's hour angle, H. The places of the Sun required for this purpose, which need be accurate only to $0'.1$, were interpolated from the extended tables circulated by Professor AIRY, using the formulæ already quoted in § 6. The position angle, p'' , is not corrected for refraction.

Applying the corrections for refraction to s'' and p'' , we shall have the values of these quantities to be compared with theory.

The computations were all made on printed forms, and were all executed in duplicate by two independent computers, Mr. WILLIAM F. McK. RITTER and Dr. WILLIAM W. TOWNSEND, and both sets are preserved in connection with the records.

WLADIWOSTOK.

No. of Photo.	Greenwich Sid. Time.			Distance of Centers.		Corr. for Refr.	ω on Plate.	Sun's Hour Angle H.	Parallactic Angle q'	Position Angle p''	Corr. for Refr.				
				Plate.	s''										
6	h	m	s	d	"	"	'	'	'	'	'				
	8	1	32.7 + $\delta \lambda_1$	98.179	869.36	+ 1.02	+218	31.0	- 3	31.0	- 2	48.6	+ 36	27.1	- 2.4
7	8	3	24.2 "	97.837	866.33	1.03	217	40.2	- 3	3.2	- 2	26.4	35	52.6	- 2.4
13	8	54	10.2 "	91.747	812.41	1.41	196	11.9	+ 9	35.9	+ 7	37.6	21	48.4	- 1.1
14	8	56	6.7 "	91.678	811.80	1.41	195	27.3	10	5.0	8	0.3	21	21.1	- 1.0
15	8	57	56.2 "	91.552	810.68	1.43	194	47.0	10	32.3	8	21.7	+ 20	56.3	- 1.0
31	10	40	59.7 "	99.013	876.75	2.24	149	51.6	36	13.4	26	40.8	- 8	31.1	+ 5.8
32	10	42	53.1 "	99.364	879.84	2.26	149	7.5	36	41.7	26	59.0	- 8	57.8	.6.2
33	10	44	37.6 "	99.492	880.99	2.27	148	16.2	37	7.7	27	15.2	- 9	33.6	6.2
34	10	46	23.2 "	100.040	885.84	2.29	147	39.5	37	34.1	27	31.4	- 9	53.7	6.5
35	10	48	5.6 "	100.226	887.49	2.33	146	52.1	37	59.6	27	47.1	- 10	25.2	6.6
36	10	49	53.6 "	100.598	890.78	2.35	146	14.1	38	26.5	28	3.6	- 10	46.2	6.9
37	10	51	27.9 "	100.895	893.41	2.36	145	29.0	38	50.0	28	17.8	- 11	16.2	6.9
38	10	53	18.9 "	101.164	895.79	+ 2.39	+144	55.6	+ 39	17.7	+ 28	34.6	- 11	32.2	+ 7.2

NAGASAKI.

No. of Photo.	Greenwich Sid. Time.			Distance of Centers.		Corr. for Refr.	ω on Plate.	Sun's Hour Angle H.	Parallactic Angle q'	Position Angle p''	Corr. for Refr.		
				Plate.	s''								
	h	m	s	d	"	"	°	'	°	'	°	'	
25	7	25	9.4	+ $\delta \lambda_2$	106.242	943.17	+ 0.43	234 57.8	- 14 35.3	- 14 35.4	+ 45 7.4	- 1.1	
26	7	25	40.6	"	105.674	938.13	0.43	234 52.7		27.5	28.0	45 6.9	- 1.1
28		26	35.2	"	105.589	937.38	0.41	234 31.2	14 13.9	14 14.8	44 54.6	- 1.1	
30		27	51.9	"	105.373	935.46	0.43	233 58.0	13 54.8	13 56.4	44 34.1	- 1.1	
31		28	23.8	"	105.260	934.46	0.43	233 44.9		46.8	48.7	44 27.0	- 1.1
33		30	25.2	"	104.682	929.33	0.44	232 46.2		16.6	19.4	43 48.6	- 1.1
34		31	5.9	"	104.540	928.06	0.44	232 30.0	13 6.4	13 9.5		39.3	- 1.1
35		33	12.7	"	104.077	923.96	0.44	231 38.9	12 34.8	12 38.8		9.9	- 1.1
36		33	52.8	"	103.834	921.80	0.45	231 28.4		24.8	12 29.0	43 6.1	- 1.1
37		34	23.6	"	103.840	921.85	0.45	231 12.0	12 17.2	12 21.5	42 54.8	- 1.1	
38		36	16.6	"	103.392	917.87	0.46	230 32.2	11 49.0	11 53.9	42 34.6	- 1.1	
44		42	12.6	"	102.161	906.94	0.46	228 7.1	10 20.2	10 26.5	41 9.3	- 1.1	
45		43	44.1	"	101.695	902.81	0.47	227 31.2	9 57.4	9 3.9	40 48.8	- 1.1	
46		44	22.7	"	101.475	900.86	0.47	227 7.8		47.8	9 54.3	40 31.7	- 1.1
48		45	52.4	"	101.108	897.60	0.48	226 36.5		25.5	32.1	40 15.8	- 1.1
49		47	21.0	"	100.992	896.57	0.48	226 4.0	9 3.4		10.1	39 56.9	- 1.1
50		48	1.1	"	100.833	895.16	0.48	225 38.3	8 53.4	9 0.2	39 38.9	- 1.1	
51		50	45.1	"	100.296	890.39	0.49	224 32.9	8 12.5	8 19.3	39 1.6	- 1.1	
52		52	31.9	"	99.921	887.06	0.50	223 51.4	7 45.9	7 52.6	38 37.9	- 1.1	
53		54	1.3	"	99.820	886.16	0.50	223 18.3		23.6	7 30.3	19.7	- 1.1
54		55	0.1	"	99.468	883.04	0.51	222 53.2	7 9.0	7 15.6	38 4.5	- 1.1	
55	7	56	10.6	"	99.223	880.86	0.51	222 24.1	6 51.4	6 57.8	37 47.4	- 1.1	
57	8	1	12.8	"	98.267	872.38	0.52	220 17.8	5 36.1	5 41.8	36 31.7	- 1.1	
58		2	58.6	"	97.864	868.80	0.52	219 23.1	5 9.7	5 15.2	35 54.9	- 1.0	
59		4	28.9	"	97.848	868.66	0.53	218 54.7	4 47.2	4 52.4	35 41.3	- 1.0	
61		7	25.3	"	97.574	866.22	0.54	217 34.5	4 3.2	4 7.8	34 50.9	- 1.0	
62		8	7.9	"	96.987	861.01	0.55	217 14.3	3 52.6	3 57.0	34 37.8	- 1.0	
63		9	1.6	"	97.090	861.93	0.55	216 52.0		39.2	3 43.4	34 24.9	- 1.0
64		9	40.8	"	96.972	860.88	0.55	216 38.0	3 29.4	3 33.4	34 17.5	- 1.0	
65		13	9.4	"	96.625	857.80	0.56	215 1.2	2 37.5	2 40.6	33 15.4	- 1.0	
67		20	3.1	"	95.339	846.38	0.58	212 4.6	- 0 54.4	- 0 55.4	31 28.8	- 0.9	
72		38	46.6	"	92.912								
74		44	11.3	"	92.587	821.95	0.66	201 20.6	+ 5 6.6	+ 5 12.0	24 47.3	- 0.6	
75		45	38.5	"	92.703	822.98	0.66	200 49.0	5 28.3	5 34.0	24 30.7	- 0.6	
76		52	14.1	"	92.017	816.89	0.68	197 38.3	7 6.9	7 13.5	22 25.8	- 0.4	
77		54	0.9	"	91.952	816.31	0.68	196 38.5		33.5	40.2	21 44.0	- 0.4
78		54	32.0	"	91.993	816.68	0.68	196 34.3		41.3	48.0	21 44.9	- 0.4
79		55	17.3	"	91.773	814.72	0.69	196 9.6	7 52.6	7 59.3	21 28.0	- 0.4	
80		55	52.0	"	91.866	815.55	0.69	195 58.8	8 1.2	8 8.0	21 23.1	- 0.4	
81	8	56	23.1	"	91.720	814.25	0.69	195 46.2	8 9.0	8 15.8	21 15.6	- 0.4	
84	9	6	7.6	"	91.457	811.92	0.72	191 14.0	10 34.7	10 40.7	18 21.9	- 0.1	
93		51	55.6	"	92.439	820.64	0.80	169 56.3	21 59.6	21 28.4	4 50.9	+ 1.0	
94		52	24.9	"	92.338	819.74	0.80	169 41.6	22 6.9	21 35.0	4 41.0	1.0	
95		53	8.9	"	92.468	820.89	0.80	169 19.2	22 17.8	21 44.8	4 26.2	1.0	
96		53	35.8	"	92.602	822.08	0.81	169 7.6	22 24.6	21 50.7	4 19.3	1.0	
99		59	3.9	"	93.061	826.16	+ 0.82	166 42.3	+ 23 46.3	+ 23 3.0	+ 2 50.0	+ 1.2	

PEKING.

No. of Photo.	Greenwich Sid. Time.			Distance of Centers.		Corr. for Refr.	ω on Plate.		Sun's Hour Angle H.		Parallactic Angle q'		Position Angle p''		Corr. for Refr.
				Plate.	s''										
15	h	m	s	d	"	"	o	'	o	'	o	'	o	'	'
	7	24	54.1 + $\delta\lambda_3$	108.895	943.02	+ 0.49	242	35.2	- 28	4.8	- 22	55.4	+ 45	11.8	- 2.3
19		31	36.3	107.183	928.19	0.51	240	5.2	- 26	24.6	21	41.4	43	45.8	- 2.4
21		34	33.6	106.432	921.68	0.52	239	4.1	- 25	40.4	21	8.4	43	12.3	- 2.4
22	7	36	8.3	105.975	917.73	0.53	238	20.1	- 25	16.8	- 20	50.7	+ 42	43.0	- 2.4
44	10	40	21.3	101.007	874.76	1.20	159	14.0	+ 20	38.1	+ 17	15.8	- 8	4.3	+ 2.0
45		41	26.4	101.167	876.15	1.21	158	45.6	20	54.3	17	28.6	- 8	23.0	2.0
46		42	32.8	101.550	879.47	1.20	158	22.6	21	10.8	17	41.6	- 8	35.4	2.1
49		46	38.7	102.061	883.89	1.21	156	37.0	22	12.1	18	29.4	- 9	43.2	2.2
50		47	53.8	102.570	888.30	1.21	156	9.6	22	30.8	18	43.8	- 9	58.6	2.2
51		49	5.7	102.838	890.62	1.22	155	40.8	22	48.8	18	57.7	- 10	16.6	2.3
53		51	43.3	103.381	895.33	1.22	154	35.8	23	28.0	19	28.0	- 10	57.2	2.4
54		52	57.2	103.329	894.87	1.22	154	4.0	23	46.5	19	42.0	- 11	16.6	2.4
56		55	22.8	103.986	900.57	1.22	153	10.0	24	22.8	20	9.6	- 11	48.0	2.5
57		56	40.0	104.308	903.35	1.22	152	39.0	24	42.0	20	24.2	- 12	7.3	2.6
58		57	57.5	104.482	904.86	1.23	152	5.9	25	1.3	20	38.9	- 12	27.7	2.6
59	10	59	2.2	104.726	906.97	1.22	151	42.6	25	17.4	20	51.0	- 12	41.1	2.7
60	11	0	11.6	105.053	909.80	1.23	151	16.1	25	34.7	21	4.0	- 12	56.6	2.8
61		1	26.0	105.451	913.25	1.23	150	43.1	25	53.3	21	17.9	- 13	18.0	2.8
63		3	50.6	105.994	917.96	1.23	149	49.8	26	29.3	21	44.8	- 13	48.7	2.8
65		6	1.6	106.356	921.09	1.23	148	58.7	27	2.0	22	9.0	- 14	19.0	2.9
67		8	28.6	107.199	928.39	1.23	147	55.7	27	38.6	22	35.9	- 14	58.6	3.0
68		9	28.7	107.136	927.85	1.23	147	38.9	27	53.6	22	47.0	- 15	6.1	3.0
69		10	55.5	107.589	931.77	1.24	147	10.0	28	15.2	23	2.8	- 15	21.6	3.2
70		11	58.0	107.722	932.92	1.24	146	40.8	28	30.8	23	14.2	- 15	39.8	3.2
71		12	58.2	108.259	937.57	1.24	146	17.3	28	45.8	23	25.1	- 15	55.2	3.2
72	11	13	59.4	108.570	940.26	+ 1.24	146	3.1	+ 29	1.0	+ 23	36.2	- 15	59.0	+ 3.3

KERGUELEN.

No. of Photo.	Greenwich Sid. Time.			Distance of Centers.		Corr. for Refr.	ω on Plate.		Sun's Hour Angle H.		Parallactic Angle q'		Position Angle p''		Corr. for Refr.
				Plate.	s''										
7	h	m	s	d	"	"	o	'	o	'	o	'	o	'	'
	7	56	12.8 + $\delta\lambda_4$	103.827	915.25	+ 0.87	- 156	34.6	- 66	38.6	224	56.4	+ 37	3.8	- 0.1
17	10	5	28.3	97.824	863.75	+ 0.34	184	22.5	- 34	25.6	216	52.4	2	4.0	+ 0.1
18	10	9	12.5	98.684	869.92	+ 0.33	185	1.6	- 33	29.8	216	20.4	1	13.4	0.1
19	10	10	23.6	98.694	870.01	+ 0.33	185	22.5	- 33	12.1	216	9.9	0	48.5	0.1
20	10	12	4.7	98.637	869.50	+ 0.33	185	52.8	- 32	46.9	215	54.8	+ 0	13.9	0.1
25	10	38	13.7	102.574	904.21	+ 0.32	191	10.1	- 26	15.8	211	22.2	- 6	17.4	0.1
32	10	56	2.0	105.559	930.52	+ 0.32	194	36.3	- 21	49.6	207	31.3	- 10	34.4	0.1
33	11	25	28.0	111.971	987.04	+ 0.31	- 199	39.2	- 14	29.4	199	43.2	- 17	0.8	+ 0.1

HOBART TOWN.

No. of Photo.	Greenwich Sid. Time.			Distance of Centers.		Corr. for Refr.	ω on Plate.	Sun's Hour Angle H.	Parallactic Angle q'	Position Angle β''	Corr. for Refr.				
				Plate.	s''										
	h	m	s	d	"	"	°	'	°	'	°	'			
9	9	59	2.6 + $\delta\lambda_6$	97.257	856.66	+ 0.28	-166	26.7	+ 41	13.4	130	34.8	+ 2	34.5	- 0.1
10	9	59	37.3	97.326	857.27	0.28	166	43.8	41	22.1	130	31.9	2	15.3	- 0.1
11	10	0	16.7	97.246	856.56	0.28	166	53.0	41	31.9	130	28.6	2	3.8	- 0.1
12		0	45.0	97.212	856.26	0.28	166	48.7	41	39.0	130	26.3	2	6.6	- 0.1
13		2	7.1	97.465	858.50	0.29	167	13.6	41	59.4	130	19.6	1	36.8	- 0.1
14		3	51.3	97.790	861.36	0.29	167	33.5	42	25.4	130	11.3	1	5.8	- 0.1
15		4	38.7	97.635	860.00	0.29	167	37.8	42	37.2	130	7.6	0	58.6	- 0.1
16		6	14.7	98.026	863.44	0.29	167	57.3	43	1.1	130	0.4	0	31.3	- 0.1
17		6	32.3	97.937	862.65	0.29	167	59.0	43	5.5	129	59.0	0	28.4	- 0.1
18		7	57.1	98.291	865.77	0.29	168	15.1	43	26.6	129	52.8	0	6.8	- 0.1
19		8	27.1	98.227	865.21	0.29	168	15.7	43	34.1	129	50.6	+ 0	4.2	- 0.1
22		14	9.7	98.943	871.51	0.31	169	24.0	44	59.5	129	27.2	- 1	31.9	- 0.1
23		14	54.0	98.897	871.11	0.31	169	34.7	45	10.6	129	24.4	- 1	45.3	- 0.1
24		15	33.3	99.125	873.12	0.31	169	46.0	45	20.4	129	21.9	- 1	59.3	- 0.1
25		16	18.7	99.321	874.84	0.31	169	56.6	45	31.7	129	19.0	- 2	12.6	- 0.1
26		17	6.1	99.486	876.30	0.32	169	55.6	45	43.5	129	16.1	- 2	14.9	- 0.1
27		18	48.2	99.366	875.24	0.32	170	8.4	46	8.9	129	10.0	- 2	37.0	- 0.1
28		19	40.8	99.562	876.97	0.32	170	31.1	46	22.0	129	6.9	- 3	3.0	- 0.1
29		20	38.5	99.834	879.36	0.32	170	40.2	46	36.4	129	3.6	- 3	15.9	- 0.1
30		21	12.5	99.998	880.81	0.32	170	50.2	46	44.9	129	1.6	- 3	27.8	- 0.1
32		22	53.6	100.398	884.33	0.33	171	14.4	47	10.1	128	56.0	- 3	58.7	- 0.1
33		23	12.2	100.101	881.71	0.33	171	5.8	47	14.7	128	55.0	- 3	51.2	- 0.1
34		23	53.5	100.251	883.03	0.33	171	16.4	47	25.0	128	52.8	- 4	4.4	- 0.1
35		24	31.2	100.472	884.98	0.33	171	21.2	47	34.4	128	50.8	- 4	11.7	- 0.1
36		25	23.6	100.537	885.56	0.34	171	43.3	47	47.5	128	48.0	- 4	37.1	- 0.1
37		25	46.4	100.627	886.35	0.34	171	31.9	47	53.2	128	46.9	- 4	27.2	- 0.2
38		26	49.6	100.849	888.30	0.34	171	52.6	48	8.9	128	43.7	- 4	52.1	- 0.2
39		27	47.7	101.079	890.33	0.34	171	58.6	48	23.4	128	40.8	- 5	2.2	- 0.2
40		28	30.4	100.877	888.55	0.34	172	15.0	48	34.0	128	38.7	- 5	21.4	- 0.2
41		29	55.3	101.065	890.20	0.35	172	15.6	48	55.2	128	34.6	- 5	36.7	- 0.2
42		30	40.6	101.516	894.19	0.35	172	19.7	49	6.5	128	32.5	- 5	43.7	- 0.2
43		31	21.2	101.370	892.89	0.35	172	35.2	49	16.6	128	30.7	- 6	1.7	- 0.2
44		32	9.8	101.845	897.08	0.35	172	43.2	49	28.7	128	28.5	- 6	12.8	- 0.2
45		32	49.2	101.616	895.06	0.35	172	47.6	49	38.5	128	26.7	- 6	19.8	- 0.2
46		33	47.1	101.995	898.40	0.36	173	1.9	49	53.0	128	24.2	- 6	37.8	- 0.2
47		34	44.4	102.142	899.69	0.36	173	4.0	50	7.2	128	21.8	- 6	43.6	- 0.2
48	10	35	44.3	102.220	900.38	+ 0.36	-173	15.2	+ 50	22.2	128	19.3	- 6	58.5	- 0.2

CAMPBELLTOWN.

No. of Photo.	Greenwich Sid. Time.			Distance of Centers.		Corr. for Refr.	ω on Plate.		Sun's Hour Angle H.		Parallactic Angle g' .		Position Angle p'' .		Corr. for Refr.
				Plate.	s''										
10	h	m	s	d	"	"	o	'	o	'	o	'	o	'	'
	9	29	3.0 + $\delta \lambda_6$	95.087	842.53	+ 0.25	159	50.5	+ 33	54.8	132	12.2	+ 10	47.1	0.0
11		32	30.3 "	95.008	841.83	0.25	160	32.4	34	46.5	131	45.6	9	49.0	0.0
12		33	4.2 "	95.105	842.69	0.25	160	35.6	34	55.9	131	41.0	9	40.7	0.0
17		42	8.7 "	95.465	845.88	0.26	162	31.7	37	10.6	130	38.8	7	10.4	0.0
18		43	6.4 "	95.244	843.92	0.26	162	50.0	37	25.0	130	32.7	6	47.5	0.0
19		43	58.8 "	95.323	844.62	0.26	162	48.1	37	38.0	130	27.2	6	45.4	0.0
23		50	35.4 "	95.898	849.71	0.27	164	15.8	39	16.9	129	48.3	4	46.6	0.0
24	9	51	27.3 "	95.892	849.66	0.27	164	12.9	39	29.8	129	43.5	4	45.5	0.0
28	10	7	44.9 "	97.843	866.95	0.29	167	35.5	43	33.5	128	25.7	+ 0	9.5	- 0.1
29		11	37.6 "	98.151	869.68	0.30	168	14.6	44	31.5	128	10.3	- 0	47.9	- 0.1
31		16	45.9 "	99.011	877.30	0.32	169	17.4	45	48.4	127	51.7	- 2	15.1	- 0.1
32		18	2.0 "	99.047	877.61	0.32	169	22.4	46	7.3	127	47.4	- 2	26.3	- 0.1
33		19	55.0 "	99.355	880.35	0.32	170	6.0	46	35.5	127	41.2	- 3	17.5	- 0.1
34		22	1.4 "	99.598	882.50	0.32	170	15.0	47	7.0	127	34.6	- 3	36.8	- 0.1
35		24	44.2 "	100.076	886.73	0.33	170	40.0	47	47.6	127	26.6	- 4	12.6	- 0.1
36		26	46.4 "	100.079	886.76	0.34	171	2.6	48	18.0	127	20.8	- 4	45.0	- 0.2
37		27	54.3 "	100.536	890.81	0.34	171	24.3	48	34.9	127	17.7	- 5	12.2	- 0.2
38		28	44.1 "	100.594	891.32	0.34	171	26.8	48	47.4	127	15.5	- 5	18.6	- 0.2
39		30	29.2 "	100.730	892.53	0.34	171	44.2	49	13.5	127	11.0	- 5	44.4	- 0.2
40		31	49.0 "	101.325	897.80	0.35	172	3.3	49	33.4	127	7.6	- 6	10.1	- 0.2
41		32	48.6 "	101.271	897.32	0.35	172	16.0	49	48.3	127	5.2	- 6	27.6	- 0.2
42		34	0.4 "	101.509	899.43	0.36	172	13.7	50	6.2	127	2.4	- 6	30.8	- 0.2
44		36	18.8 "	101.998	903.76	0.36	172	46.0	50	40.7	126	57.2	- 7	14.6	- 0.2
45		37	47.4 "	102.169	905.28	0.37	172	54.8	51	2.8	126	54.0	- 7	30.5	- 0.2
46		39	4.4 "	102.483	908.05	0.37	173	9.9	51	22.0	126	51.4	- 7	51.8	- 0.2
47		40	44.9 "	102.627	909.33	0.38	173	23.6	51	47.0	126	48.1	- 8	14.2	- 0.2
48	10	59	53.5 "	106.884	947.06	0.46	176	26.6	56	33.3	126	20.7	- 12	49.9	- 0.3
49	11	2	2.7 "	107.254	950.34	0.47	176	54.3	57	5.5	126	18.7	- 13	28.0	- 0.3
50	11	3	5.0 "	107.407	951.69	0.47	176	56.1	57	21.0	126	17.9	- 13	35.0	- 0.3
51	11	4	11.6 "	107.516	952.66	0.48	176	56.9	57	37.6	126	17.0	- 13	41.1	- 0.3
52	11	5	2.2 "	107.675	954.06	0.48	177	6.9	57	50.2	126	16.4	- 13	55.4	- 0.3
53	11	6	6.2 "	107.934	956.36	+ 0.49	177	21.4	+ 58	6.2	126	15.6	- 14	15.2	- 0.4

QUEENSTOWN.

No. of Photo.	Greenwich Sid. Time.			Distance of Centers.		Corr. for Refr.	ω on Plate.	Sun's Hour Angle H.	Parallactic Angle q'	Position Angle p''	Corr. for Refr.				
				Plate.	s''										
	h	m	s	d	"	"	°	'	°	'	°	'	°	'	
115	7	35	41.4	105.465	932.31	+ 0.25	132	53.5	26	49.6	141	57.9	40	47.1	+ 0.1
114		37	34.0	105.174	929.74	0.25	133	7.4	27	17.7	141	35.3	40	26.3	0.1
116		39	17.3	104.733	925.84	0.25	133	25.4	27	43.4	141	15.1	40	2.8	0.1
117		40	23.7	104.679	925.36	0.25	133	38.1	28	0.0	141	2.4	39	45.7	0.1
118		42	26.0	104.582	924.50	0.25	133	56.2	28	30.5	140	39.3	39	16.8	0.1
119		43	52.0	103.930	918.74	0.25	134	12.3	28	51.9	140	23.4	38	58.7	0.1
120		46	32.5	103.418	914.21	0.25	134	51.9	29	31.9	139	54.6	38	8.8	0.1
122		51	23.2	102.409	905.29	0.24	135	36.6	30	44.4	139	4.7	37	7.2	0.1
123		53	26.0	102.382	905.05	0.24	135	57.1	31	15.0	138	44.6	36	38.5	0.1
124		55	44.3	101.644	898.53	0.24	136	20.9	31	49.4	138	22.6	36	5.4	0.1
125	7	57	50.7	101.446	896.78	0.24	136	49.8	32	20.9	138	3.0	35	28.2	0.1
126	8	0	38.4	100.861	891.61	0.24	137	17.4	33	2.8	137	37.9	34	49.4	0.1
127		3	1.8	100.367	887.24	0.24	137	45.2	33	38.5	137	17.1	34	14.5	0.1
128		5	4.3	100.389	887.44	0.24	138	10.3	34	9.0	137	0.0	33	41.4	0.1
129		7	15.1	100.019	884.17	0.24	138	31.0	34	41.6	136	42.2	33	12.0	0.1
130		10	30.5	99.322	878.00	0.24	139	11.5	35	30.3	136	16.5	32	20.7	0.1
131		12	0.4	98.922	874.47	0.24	139	26.6	35	52.7	136	5.1	31	59.4	0.1
132		13	49.9	99.090	875.95	0.24	139	53.7	36	20.0	135	51.5	31	25.1	0.1
133		15	21.0	98.850	873.83	0.24	140	13.5	36	42.7	135	40.4	30	59.2	0.1
134		17	6.4	98.482	870.58	0.24	140	27.5	37	9.0	135	27.9	30	38.0	0.1
135		18	49.6	98.164	867.77	0.24	140	45.5	37	34.7	135	16.0	30	13.0	+ 0.1
142		45	21.2	95.712	846.09	0.24	146	21.0	44	11.4	132	46.2	22	55.4	0.0
139		46	36.8	95.892	847.68	0.24	146	44.6	44	30.3	132	40.5	22	27.4	0.0
144		47	38.1	95.633	845.39	0.24	146	48.0	44	45.5	132	36.0	22	18.9	0.0
143		48	41.0	95.665	845.68	0.24	147	7.9	45	1.2	132	31.5	21	54.6	0.0
145	8	50	28.9	95.446	843.74	0.25	147	24.2	45	28.1	132	23.9	21	30.7	0.0
151	9	43	19.4	95.766	846.57	0.33	158	39.4	58	38.3	130	10.4	6	32.2	- 0.3
153		43	38.7	95.934	848.05	0.34	158	55.4	58	43.1	130	10.1	6	14.8	- 0.3
154		44	31.1	95.860	847.40	0.34	158	46.5	58	56.2	130	9.1	6	19.8	- 0.3
155		46	6.6	95.996	848.60	0.34	159	11.4	59	20.0	130	7.5	5	47.7	- 0.4
156		49	32.0	96.172	850.16	0.36	159	57.1	60	11.2	130	4.4	4	46.6	- 0.4
158		52	6.1	96.661	854.48	0.36	160	29.9	60	49.6	130	1.9	4	2.3	- 0.4
159		53	35.0	96.605	853.99	0.36	160	29.8	61	11.8	130	1.4	3	55.6	- 0.4
160		55	35.1	96.526	853.29	0.38	161	1.8	61	41.7	130	0.2	3	14.5	- 0.4
161	9	56	15.8	96.674	854.60	0.38	161	10.8	61	51.8	129	59.8	3	2.6	- 0.5
163	10	6	2.5	98.309	869.06	0.44	163	19.4	64	18.0	129	56.5	+	0 10.9	- 0.6
164		7	41.4	97.966	866.02	0.44	163	43.8	64	42.7	129	56.4	-	0 22.0	- 0.6
165		8	30.9	98.151	867.65	0.44	163	37.1	64	55.0	129	56.3	-	0 19.2	- 0.6
166		9	52.3	98.308	869.04	0.45	163	46.2	65	15.3	129	56.3	-	0 34.4	- 0.6
167		11	7.0	98.822	873.58	0.46	164	8.3	65	34.0	129	56.4	-	1 2.2	- 0.6
171		18	18.7	99.475	879.36	0.51	165	18.0	67	21.5	129	57.8	-	2 47.3	- 0.7
172		21	26.7	100.096	884.85	0.53	165	57.3	68	8.4	129	59.0	-	3 41.1	- 0.8
173		23	11.1	100.142	885.25	0.53	166	20.6	68	34.4	129	59.8	-	4 12.5	- 0.8
176		41	26.0	103.606	915.87	0.72	169	14.0	73	7.3	130	15.0	-	8 38.5	- 1.1
177	10	41	56.2	103.529	915.19	+ 0.73	169	12.2	+	73 14.8	130	15.6	-	8 39.1	- 1.1

CHATHAM ISLAND.

No. of Photo.	Greenwich Sid. Time.			Distance of Centers.		Corr. for Refr.	ω on Plate.	Sun's Hour Angle H.	Parallactic Angle q'	Position Angle β''	Corr. for Refr.				
				Plate.	"										
15	h	m	s	d	"	"	°	'	°	'	°	'			
	8	20	16.1 + $\delta\lambda_0$	97.265	863.88	+ 0.25	136	43.3	+ 52	36.4	129	14.5	+ 29	26.5	+ 0.1
16		21	7.4 "	97.359	864.71	0.25	136	46.2	52	49.1	129	12.7	29	19.6	0.1
17	8	22	10.7 "	97.358	864.71	0.25	137	5.7	53	4.9	129	10.6	28	55.3	+ 0.1
24	9	8	32.7 "	94.372	838.18	0.30	146	9.7	64	38.3	128	28.8	16	9.1	- 0.3
25		9	41.6 "	94.430	838.70	0.31	146	30.0	64	55.5	128	28.9	15	43.2	- 0.3
27		18	29.1 "	94.463	838.99	0.33	148	2.4	67	7.0	128	31.6	13	28.4	- 0.5
29	9	32	43.2 "	94.807	842.05	+ 0.41	150	51.4	+ 70	39.8	128	41.9	+ 9	23.0	- 0.8

§ 9. TABULAR DATA FOR COMPARISON WITH OBSERVATIONS.

All the observations of the Transit with which we have to deal in the present work may, in effect, be considered as determinations of the apparent angular distance between the center of Venus and that of the Sun, or of the angle of position of the line joining these centers. Observations of contacts may be regarded as fixing the moments at which the angular distance of centers was equal to the sum or the difference of the angular semi-diameters of the two bodies. For the purpose of comparing with observations, and deducing the final values of the required elements, it is necessary to express the distance of centers in terms of the tabular elements and of the corrections to them. The formulæ for doing this having already been discussed, it only remains to present the numerical data.

The tabular quantities required are the geocentric right ascensions, declinations, and semi-diameters of the Sun and Venus, and their equatorial horizontal parallaxes. At least four sets of such quantities have been computed and published.

The first of these in chronological order was by Dr. THEODORE VON OPPOLZER, and appeared in the *Sitzungsberichte* of the Vienna Academy of Sciences for 1870 (Vol. 61, p. 515). Here the tabular quantities are computed with great care from LE VERRIER'S tables of the Sun and Venus, and very complete tables and formulæ are given for computing the apparent distance of centers and position-angle as seen from any part of the Earth. But for the considerable magnitude of the errors of LE VERRIER'S tables of Venus at the time of the transit, no further determination of the tabular elements would have been necessary.

Secondly, in 1872 Mr. GEORGE W. HILL prepared tabular data from HANSEN'S tables of the Sun and his own tables of Venus, in a work published by the American Commission on the Transit of Venus, Part II of its papers. This paper refers mainly to times of contacts, for which very elaborate tables and formulæ are given. On page 44 are given formulæ for position-angle and distance which, however, cannot be employed in comparing with observations, because the angles refer not to the Sun's center, but to the point in which the line joining the centers of the Sun and Venus intersects the celestial sphere.

Thirdly, in 1875 Professor AIRY issued very complete tables of data, in which the tabular places of the Sun and Venus were interpolated from the *Nautical Almanac*. It is with these tables that the British observations have been compared.

Fourthly, in the Additions to the *Connaissance des Temps* for 1878 (Paris, 1876), M. PUISEUX gives a set of tabular data in which the places of the Sun and Venus are derived from LE VERRIER'S tables. Of course his geocentric elements are substantially identical with those of OPPOLZER, as the same tables were used.

In the present discussion the comparison is made with HILL'S elements, because they are those of which the errors are the smallest. The differential co-efficients of the corrections to the elements are themselves functions of the elements, and any errors of the latter will necessarily affect the former. As, however, it may be deemed necessary to compare with AIRY'S or some other elements, the following comparisons of the several elements referred to are given. In changing Washington to Greenwich time, the difference of longitude is assumed to be $5^h 8^m 12^s.1$.

HILL'S Elements.

Washington Mean Time.	Greenwich Sidereal Time.	Right Ascensions.			Declinations.		
		Venus.			The Sun.		
Dec. d h	h m s	° ' "	° ' "	° ' "	° ' "	° ' "	° ' "
8 8	6 18 39.1	255 58 56.03	255 42 16.80	— 22 38 9.96	— 22 48 24.39		
9	7 18 49.0	255 57 21.96	255 45 1.47	— 22 37 22.29	— 22 48 39.36		
10	8 18 58.9	255 55 47.90	255 47 46.15	— 22 36 34.60	— 22 48 54.28		
11	9 19 8.8	255 54 13.86	255 50 30.84	— 22 35 46.90	— 22 49 9.15		
12	10 19 18.6	255 52 39.84	255 53 15.54	— 22 34 59.18	— 22 49 23.98		
13	11 19 28.5	255 51 5.83	255 56 0.25	— 22 34 11.44	— 22 49 38.77		
14	12 19 38.4	255 49 31.85	255 58 44.98	— 22 33 23.67	— 22 49 53.51		

Comparing these with the corresponding elements published by Professors AIRY and OPPOLZER, we find the following differences for the positions of Venus relative to those of the Sun:

W. M. T.	AIRY-HILL.		AIRY-OPPOLZER.		OPPOLZER-HILL.	
	$\Delta(a-a')$	$\Delta(\delta-\delta')$	$\Delta(a-a')$	$\Delta(\delta-\delta')$	$\Delta(a-a')$	$\Delta(\delta-\delta')$
h	"	"	"	"	"	"
9	— 3.62	— 0.95	— 0.61	— 0.14	— 3.04	— 0.80
10	— 3.61	— 0.95	— 0.59	— 0.14	— 3.02	— 0.81
11	— 3.57	— 0.95	— 0.57	— 0.13	— 3.02	— 0.81
12	— 3.56	— 0.96	— 0.56	— 0.13	— 3.02	— 0.82
13	— 3.54	— 0.98	— 0.55	— 0.13	— 3.01	— 0.84

Each of these three sets of comparisons was made directly and independently, which accounts for their failure to agree rigorously. The comparison of AIRY with OPPOLZER was not actually made for the Washington hours given, but for 14^h , 15^h , etc., Paris mean time.

The constant of parallax is assumed to be $8''.848$ in the papers of HILL and of

OPPOLZER, and 8".95 in that of AIRY. To reduce our comparisons to those with AIRY'S tables, we have to put

$$\delta A = \text{between } -3''.62 \text{ and } -3''.54$$

$$\delta D = \text{between } -0''.94 \text{ and } -0''.98$$

$$\delta \pi = +0''.102$$

the proper values of δA and δD being interpolated to the time of observation.

The tabular quantities required by the theory are derived from the preceding positions of Venus and the Sun, supposing the mean solar parallax to be 8".848. The formulæ required for their construction and use have been given on pages 54 to 56.

TABLE I.

Geocentric Tabular Distances and Position-Angles of the Center of Venus from that of the Sun (derived from HILL'S positions in "Papers relating to the Transit of Venus, etc.," p. 9).

Gr. Sid. Time.	s	Diff.	p	Diff.	Gr. Sid. Time.	s	Diff.	p	Diff.
h m	"	"	° '	'	h m	"	"	° '	'
6 20	1104.051	- 2.707	+ 56 9.02	- 9.63	6 50	1026.871	- 2.419	+ 50 59.17	- 11.12
21	1101.344	2.698	55 59.37	9.67	51	1024.452	2.408	50 48.05	11.17
22	1098.646	2.689	55 49.72	9.72	52	1022.044	2.397	50 36.88	11.22
23	1095.957	2.680	55 40.00	9.76	53	1019.647	2.386	50 25.66	11.27
24	1093.277	2.672	55 30.24	9.80	54	1017.261	2.375	50 14.39	11.32
25	1090.605	2.663	+ 55 20.44	9.86	55	1014.886	2.364	+ 50 3.07	11.38
26	1087.942	2.655	55 10.58	9.91	56	1012.522	2.353	49 51.69	11.43
27	1085.287	2.645	55 0.67	9.95	57	1010.169	2.342	49 40.26	11.48
28	1082.642	2.637	54 50.72	10.00	58	1007.827	2.330	49 28.78	11.54
29	1080.005	- 2.627	54 40.72	- 10.05	59	1005.497	- 2.320	49 17.24	- 11.58
6 30	1077.378	2.617	+ 54 30.67	10.10	7 0	1003.177	2.308	+ 49 5.66	11.64
31	1074.761	2.608	54 20.57	10.15	1	1000.869	2.296	48 54.02	11.70
32	1072.153	2.599	54 10.42	10.20	2	998.573	2.285	48 42.32	11.75
33	1069.554	2.590	54 0.22	10.24	3	996.288	2.273	48 30.57	11.81
34	1066.964	2.581	53 49.98	10.29	4	994.015	2.261	48 18.76	11.86
35	1064.383	2.571	+ 53 39.69	10.34	5	991.754	2.249	+ 48 6.90	11.91
36	1061.812	2.561	53 29.35	10.39	6	989.505	2.237	47 54.99	11.97
37	1059.251	2.551	53 18.96	10.45	7	987.268	2.225	47 43.02	12.02
38	1056.700	2.542	53 8.51	10.50	8	985.043	2.213	47 31.00	12.08
39	1054.158	- 2.531	52 58.01	- 10.55	9	982.830	- 2.202	47 18.92	- 12.13
6 40	1051.627	2.521	+ 52 47.46	10.60	7 10	980.628	2.189	+ 47 6.79	12.18
41	1049.106	2.510	52 36.86	10.65	11	978.439	2.177	46 54.61	12.24
42	1046.596	2.501	52 26.21	10.70	12	976.262	2.164	46 42.37	12.29
43	1044.095	2.490	52 15.51	10.75	13	974.098	2.152	46 30.08	12.35
44	1041.605	2.481	52 4.76	10.80	14	971.946	2.140	46 17.73	12.40
45	1039.124	2.471	+ 51 53.96	10.86	15	969.806	2.127	+ 46 5.33	12.46
46	1036.653	2.461	51 43.10	10.91	16	967.679	2.114	45 52.87	12.51
47	1034.192	2.451	51 32.19	10.96	17	965.565	2.102	45 40.36	12.57
48	1031.741	2.440	51 21.23	11.00	18	963.463	2.089	45 27.79	12.62
49	1029.301	- 2.430	51 10.23	- 11.06	19	961.374	- 2.076	45 15.17	- 12.67
6 50	1026.871		+ 50 59.17		7 20	959.298		+ 45 2.50	

TABLE I—Continued.

Gr. Sid. Time.	<i>s</i>	Diff.	<i>p</i>	Diff.	Gr. Sid. Time.	<i>s</i>	Diff.	<i>p</i>	Diff.
h m	"	"	° ' "	' "	h m	"	"	° ' "	' "
7 20	959.298	- 2.062	+ 45 2.50	- 12.73	8 7	877.968	- 1.353	+ 34 6.29	- 15.18
21	957.236	2.048	44 49.77	12.78	8	876.615	1.336	33 51.11	15.23
22	955.188	2.036	44 36.99	12.84	9	875.279	1.319	33 35.88	15.28
23	953.152	2.023	44 24.15	12.90	8 10	873.960	1.301	+ 33 20.60	15.32
24	951.129	2.009	44 11.25	12.95	11	872.659	1.284	33 5.28	15.37
25	949.120	1.996	+ 43 58.30	13.00	12	871.375	1.266	32 49.91	15.41
26	947.124	1.983	43 45.30	13.06	13	870.109	1.248	32 34.50	15.46
27	945.141	1.969	43 32.24	13.11	14	868.861	1.231	32 19.04	15.51
28	943.172	1.955	43 19.13	13.16	15	867.630	1.213	+ 32 3.53	15.55
29	941.217	- 1.941	43 5.97	- 13.22	16	866.417	1.195	31 47.98	15.59
7 30	939.276	1.927	+ 42 52.75	13.28	17	865.222	1.177	31 32.39	15.63
31	937.349	1.913	42 39.47	13.33	18	864.045	1.159	31 16.76	15.68
32	935.436	1.899	42 26.14	13.39	19	862.886	- 1.142	31 1.08	- 15.72
33	933.537	1.885	42 12.75	13.44	8 20	861.744	1.124	+ 30 45.36	15.76
34	931.652	1.870	41 59.31	13.49	21	860.620	1.106	30 29.60	15.80
35	929.782	1.856	+ 41 45.82	13.55	22	859.514	1.088	30 13.80	15.84
36	927.926	1.841	41 32.27	13.60	23	858.426	1.069	29 57.96	15.87
37	926.085	1.827	41 18.67	13.66	24	857.357	1.051	29 42.09	15.92
38	924.058	1.812	41 5.01	- 13.71	25	856.306	1.032	+ 29 26.17	15.96
39	922.446	- 1.798	40 51.30	13.76	26	855.274	1.014	29 10.21	15.99
7 40	920.648	1.783	+ 40 37.54	13.81	27	854.260	0.995	28 54.22	16.03
41	918.865	1.768	40 23.73	13.87	28	853.265	0.977	28 38.19	16.07
42	917.097	1.754	40 09.86	13.92	29	852.288	- 0.958	28 22.12	- 16.11
43	915.343	1.739	39 55.94	13.98	8 30	851.330	0.940	+ 28 6.01	16.14
44	913.604	1.724	39 41.96	14.04	31	850.390	0.922	27 49.87	16.18
45	911.880	1.708	+ 39 27.92	14.08	32	849.468	0.903	27 33.69	16.21
46	910.172	1.693	39 13.84	14.13	33	848.565	0.884	27 17.48	16.24
47	908.479	1.677	38 59.71	14.19	34	847.681	0.865	27 1.24	16.28
48	906.802	1.662	38 45.52	14.24	35	846.816	0.845	+ 26 44.96	16.31
49	905.140	- 1.646	38 31.28	- 14.29	36	845.971	0.826	26 28.65	16.34
7 50	903.494	1.631	+ 38 16.99	14.35	37	845.145	0.807	26 12.31	16.38
51	901.863	1.615	38 2.64	14.40	38	844.338	0.788	25 55.93	16.41
52	900.248	1.599	37 48.24	14.45	39	843.550	- 0.768	25 39.52	- 16.44
53	898.649	1.583	37 33.79	14.49	8 40	842.782	0.749	+ 25 23.08	16.47
54	897.066	1.568	37 19.30	14.55	41	842.033	0.730	25 6.61	16.50
55	895.498	1.551	+ 37 4.75	14.60	42	841.303	0.711	24 50.11	16.52
56	893.947	1.535	36 50.15	14.65	43	840.592	0.692	24 33.59	16.55
57	892.412	1.519	36 35.50	14.70	44	839.900	0.672	24 17.04	16.57
58	890.893	1.503	36 20.80	14.74	45	839.228	0.652	+ 24 0.47	16.60
59	889.390	- 1.486	36 6.06	- 14.80	46	838.576	0.633	23 43.87	16.63
8 0	887.904	1.469	+ 35 51.26	14.85	47	837.943	0.613	23 27.24	16.66
1	886.435	1.453	35 36.41	14.90	48	837.330	0.594	23 10.58	16.68
2	884.982	1.436	35 21.51	14.95	49	836.736	- 0.574	22 53.90	- 16.70
3	883.546	1.419	35 6.56	15.00	8 50	836.162	0.555	+ 22 37.20	16.73
4	882.127	1.403	34 51.56	15.04	51	835.607	0.536	22 20.47	16.75
5	880.724	1.386	+ 34 36.52	15.09	52	835.071	0.516	22 3.72	16.77
6	879.338	- 1.370	34 21.43	- 15.14	53	834.555	- 0.496	21 46.95	- 16.78
7	877.968		34 6.29		54	834.059		+ 21 30.17	

TABLE I—Continued.

Gr. Sid. Time.	s	Diff.	p	Diff.	Gr. Sid. Time.	s	Diff.	p	Diff.
h m	"	"	o /	'	h m	"	"	o /	'
8 54	834.059	- 0.475	+ 21 30.17	- 16.81	9 41	833.518	+ 0.473	+ 8 13.07	- 16.81
55	833.584	0.455	21 13.36	16.83	42	833.991	0.492	7 56.26	16.79
56	833.129	0.435	20 56.53	16.84	43	834.483	0.512	7 39.47	16.77
57	832.694	0.415	20 39.69	16.86	44	834.995	0.531	7 22.70	16.75
58	832.279	0.396	20 22.83	16.87	45	835.526	0.551	+ 7 5.95	16.73
59	831.883	- 0.375	20 5.96	- 16.90	46	836.077	0.572	6 49.22	16.70
9 0	831.508	0.356	+ 19 49.06	16.91	47	836.649	0.591	6 32.52	16.68
1	831.152	0.335	19 32.15	16.93	48	837.240	0.610	6 15.84	16.66
2	830.817	0.316	19 15.22	16.93	49	837.850	+ 0.630	5 59.18	- 16.64
3	830.501	0.295	18 58.29	16.94	9 50	838.480	0.649	+ 5 42.54	16.61
4	830.206	0.276	18 41.35	16.96	51	839.129	0.669	5 25.93	16.58
5	829.930	0.255	+ 18 24.39	16.97	52	839.798	0.688	5 9.35	16.55
6	829.675	0.235	18 7.42	16.98	53	840.486	0.708	4 52.80	16.53
7	829.440	0.214	17 50.44	16.99	54	841.194	0.728	4 36.27	16.50
8	829.226	0.194	17 33.45	17.00	55	841.922	0.748	+ 4 19.77	16.47
9	829.032	0.174	17 16.45	- 17.01	56	842.670	0.767	4 3.30	16.44
9 10	828.858	- 0.154	+ 16 59.44	17.01	57	843.437	0.786	3 46.86	16.41
11	828.704	0.133	16 42.43	17.02	58	844.223	0.805	3 30.45	16.39
12	828.571	0.112	16 25.41	17.02	59	845.028	+ 0.824	3 14.06	- 16.35
13	828.459	0.092	16 8.39	17.03	10 0	845.852	0.843	+ 2 57.71	16.32
14	828.367	0.071	15 51.36	17.03	1	846.695	0.862	2 41.39	16.29
15	828.296	0.052	+ 15 34.33	17.03	2	847.557	0.881	2 25.10	16.25
16	828.244	0.032	15 17.30	17.03	3	848.438	0.900	2 8.85	16.22
17	828.212	- 0.012	15 0.27	17.03	4	849.338	0.920	1 52.63	16.19
18	828.200	+ 0.008	14 43.24	17.04	5	850.258	0.938	+ 1 36.44	16.15
19	828.208	+ 0.028	14 26.20	- 17.04	6	851.196	0.957	1 20.29	16.11
9 20	828.236	0.049	+ 14 9.16	17.03	7	852.153	0.975	1 4.18	16.07
21	828.285	0.069	13 52.13	17.03	8	853.128	0.993	0 48.11	16.02
22	828.354	0.090	13 35.10	17.03	9	854.121	+ 1.011	0 32.09	- 15.99
23	828.444	0.110	13 18.07	17.02	10 10	855.132	1.030	+ 0 16.10	15.96
24	828.554	0.130	13 1.05	17.02	11	856.162	1.048	+ 0 0.14	15.92
25	828.684	0.151	+ 12 44.03	17.01	12	857.210	1.067	- 0 15.78	15.88
26	828.835	0.171	12 27.02	17.00	13	858.277	1.086	0 31.66	15.85
27	829.006	0.192	12 10.02	17.00	14	859.363	1.103	0 47.51	15.81
28	829.198	0.212	11 53.02	16.99	15	860.466	1.121	- 1 3.32	15.76
29	829.410	+ 0.232	11 36.03	- 16.99	16	861.587	1.139	1 19.08	15.72
9 30	829.642	0.252	+ 11 19.04	16.97	17	862.726	1.157	1 34.80	15.68
31	829.894	0.272	11 2.07	16.96	18	863.883	1.176	1 50.48	15.64
32	830.166	0.292	10 45.11	16.95	19	865.059	+ 1.193	2 6.12	- 15.60
33	830.458	0.311	10 28.16	16.94	10 20	866.252	1.211	- 2 21.72	15.55
34	830.769	0.332	10 11.22	16.93	21	867.463	1.228	2 37.27	15.51
35	831.101	0.352	+ 9 54.29	16.91	22	868.691	1.246	2 52.78	15.47
36	831.453	0.373	9 37.38	16.89	23	869.937	1.264	3 8.25	15.42
37	831.826	0.393	9 20.49	16.88	24	871.201	1.281	3 23.67	15.38
38	832.219	0.414	9 3.61	16.86	25	872.482	1.299	- 3 39.05	15.33
39	832.633	0.433	8 46.75	16.85	26	873.781	1.316	3 54.38	15.28
9 40	833.066	+ 0.452	8 29.90	- 16.83	27	875.097	+ 1.333	4 9.66	- 15.24
41	833.518		+ 8 13.07		28	876.430		- 4 24.90	

TABLE I—Continued.

Gr. Sid. Time.	<i>s</i>	Diff.	<i>p</i>	Diff.	Gr. Sid. Time.	<i>s</i>	Diff.	<i>p</i>	Diff.
h m	"	"	° '	'	h m	"	"	° '	'
10 28	876.430	+ 1.350	— 4 24.90	— 15.19	11 15	956.965	+ 2.061	— 15 23.90	— 12.74
29	877.780	1.368	4 40.09	15.14	16	959.026	2.074	15 36.64	12.68
10 30	879.148	1.385	4 55.23	15.10	17	961.100	2.087	15 49.32	12.63
31	880.533	1.401	5 10.33	15.05	18	963.187	2.100	16 1.95	12.57
32	881.934	1.418	5 25.38	15.00	19	965.287	+ 2.113	16 14.52	— 12.52
33	883.352	1.434	5 40.38	14.96	11 20	967.400	2.125	— 16 27.04	12.46
34	884.786	1.450	5 55.34	14.90	21	969.525	2.138	16 39.50	12.41
35	886.236	1.467	— 6 10.24	14.85	22	971.663	2.151	16 51.91	12.36
36	887.703	1.483	6 25.09	14.80	23	973.814	2.163	17 4.27	12.30
37	889.186	1.500	6 39.89	14.76	24	975.977	2.175	17 16.57	12.25
38	890.686	1.517	6 54.65	14.71	25	978.152	2.188	— 17 28.82	12.19
39	892.203	+ 1.533	7 9.36	— 14.66	26	980.340	2.200	17 41.01	12.14
10 40	893.736	1.549	— 7 24.02	14.60	27	982.540	2.212	17 53.15	12.08
41	895.285	1.565	7 38.62	14.55	28	984.752	2.224	18 5.23	12.02
42	896.850	1.581	7 53.17	14.51	29	986.976	+ 2.236	18 17.25	— 11.97
43	898.431	1.598	8 7.68	14.46	11 30	989.212	2.248	— 18 29.22	11.92
44	900.029	1.613	8 22.14	14.41	31	991.460	2.260	18 41.14	11.87
45	901.642	1.629	— 8 36.55	14.35	32	993.720	2.273	18 53.01	11.81
46	903.271	1.644	8 50.90	14.29	33	995.993	2.285	19 4.82	11.76
47	904.915	1.660	9 5.19	14.25	34	998.278	2.296	19 16.58	11.71
48	906.575	1.676	9 19.44	14.20	35	1000.574	2.307	— 19 28.29	11.65
49	908.251	+ 1.691	9 33.64	— 14.14	36	1002.881	2.318	19 39.94	11.59
10 50	909.942	1.707	— 9 47.78	14.09	37	1005.199	2.329	19 51.53	11.54
51	911.649	1.722	10 1.87	14.04	38	1007.528	2.341	20 3.07	11.49
52	913.371	1.736	10 15.91	13.98	39	1009.869	+ 2.352	20 14.56	— 11.44
53	915.107	1.751	10 29.89	13.93	11 40	1012.221	2.363	— 20 26.00	11.38
54	916.858	1.766	10 43.82	13.88	41	1014.584	2.374	20 37.38	11.33
55	918.624	1.781	— 10 57.70	13.83	42	1016.958	2.385	20 48.71	11.28
56	920.405	1.796	11 11.53	13.77	43	1019.343	2.395	20 59.99	11.23
57	922.201	1.811	11 25.30	13.72	44	1021.738	2.406	21 11.22	11.18
58	924.012	1.826	11 39.02	13.66	45	1024.144	2.417	— 21 22.40	11.12
59	925.838	+ 1.840	11 52.68	— 13.60	46	1026.561	2.428	21 33.52	11.07
11 0	927.678	1.854	— 12 6.28	13.55	47	1028.989	2.439	21 44.59	11.02
1	929.532	1.869	12 19.83	13.50	48	1031.428	2.450	21 55.61	10.96
2	931.401	1.884	12 33.33	13.45	49	1033.878	+ 2.460	22 6.57	— 10.91
3	933.285	1.898	12 46.78	13.39	11 50	1036.338	2.470	— 22 17.48	10.86
4	935.183	1.913	13 0.17	13.32	51	1038.808	2.480	22 28.34	10.80
5	937.096	1.926	— 13 13.52	13.27	52	1041.288	2.490	22 39.14	10.76
6	939.022	1.940	13 26.79	13.23	53	1043.778	2.500	22 49.90	10.70
7	940.962	1.953	13 40.02	13.18	54	1046.278	2.510	23 0.60	10.66
8	942.915	1.967	13 53.20	13.12	55	1048.788	2.521	— 23 11.26	10.61
9	944.882	+ 1.980	14 6.32	— 13.07	56	1051.309	2.531	23 21.87	10.56
11 10	946.862	1.994	— 14 19.39	13.01	57	1053.840	2.541	23 32.43	10.50
11	948.856	2.007	14 32.40	12.96	58	1056.381	2.551	23 42.93	10.45
12	950.863	2.021	14 45.36	12.90	59	1058.932	+ 2.561	23 53.38	— 10.40
13	952.884	2.034	14 58.26	12.85	12 0	1061.493	2.570	— 24 3.78	10.35
14	954.918	+ 2.047	15 11.11	— 12.79	1	1064.063	+ 2.579	24 14.13	— 10.30
15	956.965	— 15 23.90	— 15 23.90		2	1066.642		24 24.43	

TABLE I—Continued.

Gr. Sid. Time.	<i>s</i>	Diff.	<i>p</i>	Diff.	Gr. Sid. Time.	<i>s</i>	Diff.	<i>p</i>	Diff.
h m	"	"	o ' /	'	h m	"	"	o ' /	'
12 2	1066.642	+ 2.588	— 24 24.43	— 10.26	12 11	1090.274	+ 2.671	— 25 54.95	— 9.81
3	1069.230	2.598	24 34.69	10.21	12	1092.945	2.680	26 4.76	9.77
4	1071.828	2.607	24 44.90	10.16	13	1095.625	2.689	26 14.53	9.72
5	1074.435	2.617	— 24 55.06	10.11	14	1098.314	2.698	26 24.25	9.67
6	1077.052	2.626	25 5.17	10.05	15	1101.012	2.706	— 26 33.92	9.62
7	1079.678	2.636	25 15.22	10.01	16	1103.718	2.715	26 43.54	9.57
8	1082.314	2.644	25 25.23	9.95	17	1106.433	2.724	26 53.11	9.53
9	1084.958	+ 2.654	25 35.18	— 9.91	18	1109.157	2.733	27 2.64	9.49
12 10	1087.612	2.662	— 25 45.09	9.86	19	1111.890	+ 2.741	27 12.13	— 9.44
11	1090.274		25 54.95		12 20	1114.631		— 27 21.57	

TABLE II.

Parallactic Elements, θ , θ' , R, R'.

Gr. Sid. Time.	θ	Diff.	θ'	Diff.	log R	Diff.	log R'	Diff.
h m	o ' /	'	o ' /	'				
6 20	149 56.4		104 7.8		1.19691		1.32162	
21	150 5.4	+ 9.0	104 13.0	+ 5.2	1.19807	+ 0.00116	1.32097	— 0.00065
22	150 14.4	9.0	104 18.2	5.2	1.19923	.00116	1.32031	.00066
23	150 23.4	9.0	104 23.5	5.3	1.20039	.00116	1.31964	.00067
24	150 32.4	9.0	104 28.8	5.3	1.20155	.00116	1.31897	.00067
25	150 41.4	9.0	104 34.1	5.3	1.20271	.00116	1.31829	.00068
26	150 50.4	9.0	104 39.4	5.3	1.20388	.00117	1.31761	.00068
27	150 59.4	9.0	104 44.8	5.4	1.20505	.00117	1.31692	.00069
28	151 8.4	9.0	104 50.2	5.4	1.20622	.00117	1.31622	.00070
29	151 17.4	9.0	104 55.6	5.4	1.20739	.00117	1.31551	.00071
6 30	151 26.4	9.0	105 1.0	5.4	1.20856	.00117	1.31479	.00072
31	151 35.4	+ 9.0	105 6.5	+ 5.5	1.20974	+ 0.00118	1.31406	— 0.00073
32	151 44.3	8.9	105 12.0	5.5	1.21092	.00118	1.31333	.00073
33	151 53.2	8.9	105 17.6	5.6	1.21210	.00118	1.31259	.00074
34	152 2.1	8.9	105 23.3	5.7	1.21328	.00118	1.31184	.00075
35	152 11.0	8.9	105 29.0	5.7	1.21446	.00118	1.31108	.00076
36	152 19.9	8.9	105 34.8	5.8	1.21564	.00118	1.31032	.00076
37	152 28.8	8.9	105 40.6	5.8	1.21682	.00118	1.30955	.00077
38	152 37.7	8.9	105 46.5	5.9	1.21800	.00118	1.30877	.00078
39	152 46.6	8.9	105 52.5	6.0	1.21918	.00118	1.30799	.00078
6 40	152 55.5	8.9	105 58.5	6.0	1.22037	.00119	1.30720	.00079
41	153 4.4	+ 8.9	106 4.6	+ 6.1	1.22156	+ 0.00119	1.30639	— 0.00081
42	153 13.3	8.9	106 10.7	6.1	1.22275	.00119	1.30558	.00081
43	153 22.2	8.9	106 16.8	6.1	1.22394	.00119	1.30476	.00082
44	153 31.1	8.8	106 23.0	6.2	1.22513	.00119	1.30393	.00083
45	153 39.9	8.8	106 29.2	6.2	1.22632	.00119	1.30310	.00083
46	153 48.7	8.8	106 35.5	6.3	1.22751	.00119	1.30226	.00084
47	153 57.5	+ 8.8	106 41.8	+ 6.3	1.22870	+ 0.00119	1.30141	— 0.00085

TABLE II—Continued.

Gr. Sid. Time.	θ	Diff.	θ'	Diff.	log R	Diff.	log R'	Diff.
h m	° '	'	° '	'				
6 47	153 57.5	+ 8.8	106 41.8	+ 6.3	1.22870	+ 0.00119	1.30141	- 0.00085
48	154 6.3	8.8	106 48.1	6.4	1.22989	.00119	1.30056	.00087
49	154 15.1	8.8	106 54.5	6.4	1.23108	.00120	1.29969	.00088
6 50	154 23.9	+ 8.9	107 0.9	+ 6.5	1.23228	+ 0.00120	1.29881	- 0.00089
51	154 32.8	8.8	107 7.4	6.5	1.23348	.00120	1.29792	.00090
52	154 41.6	8.9	107 13.9	6.6	1.23468	.00120	1.29702	.00090
53	154 50.5	8.8	107 20.5	6.7	1.23588	.00120	1.29612	.00091
54	154 59.3	8.8	107 27.2	6.8	1.23708	.00120	1.29521	.00092
55	155 8.1	8.9	107 34.0	6.9	1.23828	.00120	1.29429	.00093
56	155 17.0	8.8	107 40.9	7.0	1.23948	.00120	1.29336	.00094
57	155 25.8	8.8	107 47.9	7.1	1.24068	.00120	1.29242	.00095
58	155 34.6	8.8	107 55.0	7.1	1.24188	.00120	1.29147	.00096
59	155 43.4	8.8	108 2.1	7.1	1.24308	.00121	1.29051	.00098
7 0	155 52.2	+ 8.8	108 9.2	+ 7.2	1.24429	+ 0.00121	1.28953	- 0.00099
1	156 1.0	8.8	108 16.4	7.3	1.24550	.00120	1.28854	.00100
2	156 9.8	8.8	108 23.7	7.4	1.24670	.00120	1.28754	.00100
3	156 18.6	8.8	108 31.1	7.4	1.24790	.00120	1.28654	.00101
4	156 27.4	8.7	108 38.5	7.4	1.24910	.00120	1.28553	.00101
5	156 36.1	8.7	108 45.9	7.5	1.25030	.00120	1.28452	.00103
6	156 44.8	8.7	108 53.4	7.5	1.25150	.00120	1.28349	.00104
7	156 53.5	8.7	109 0.9	7.5	1.25270	.00120	1.28245	.00106
8	157 2.2	8.7	109 8.4	7.6	1.25390	.00120	1.28139	.00107
9	157 10.9	8.7	109 16.0	7.6	1.25510	.00120	1.28032	.00108
7 10	157 19.6	+ 8.7	109 23.6	+ 7.7	1.25630	+ 0.00120	1.27924	- 0.00109
11	157 28.3	8.7	109 31.3	7.8	1.25750	.00120	1.27815	.00110
12	157 37.0	8.6	109 39.1	7.9	1.25870	.00120	1.27705	.00111
13	157 45.6	8.6	109 47.0	8.1	1.25990	.00119	1.27594	.00112
14	157 54.2	8.6	109 55.1	8.1	1.26110	.00119	1.27482	.00113
15	158 2.8	8.6	110 3.2	8.2	1.26229	.00119	1.27369	.00114
16	158 11.4	8.6	110 11.4	8.3	1.26348	.00119	1.27255	.00115
17	158 20.0	8.6	110 19.7	8.4	1.26467	.00119	1.27140	.00116
18	158 28.6	8.6	110 28.1	8.5	1.26586	.00119	1.27024	.00117
19	158 37.2	+ 8.6	110 36.6	+ 8.6	1.26705	+ 0.00119	1.26907	- 0.00118
7 20	158 45.8	8.6	110 45.2	8.6	1.26824	.00119	1.26789	.00119
21	158 54.4	8.6	110 53.8	8.7	1.26943	.00119	1.26670	.00120
22	159 3.0	8.5	111 2.5	8.8	1.27062	.00118	1.26550	.00121
23	159 11.5	8.5	111 11.3	8.9	1.27180	.00118	1.26429	.00123
24	159 20.0	8.5	111 20.2	9.0	1.27298	.00118	1.26306	.00124
25	159 28.5	8.5	111 29.2	9.0	1.27416	.00118	1.26182	.00125
26	159 37.0	8.5	111 38.2	9.1	1.27534	.00118	1.26057	.00127
27	159 45.5	8.5	111 47.3	9.2	1.27652	.00117	1.25930	.00128
28	159 54.0	8.5	111 56.5	9.2	1.27769	.00117	1.25802	.00129
29	160 2.5	8.5	112 5.7	9.3	1.27886	.00117	1.25673	.00130
7 30	160 11.0	+ 8.5	112 15.0	+ 9.4	1.28003	+ 0.00117	1.25543	- 0.00132
31	160 19.5	8.5	112 24.4	9.5	1.28120	.00117	1.25411	.00134
32	160 28.0	8.4	112 33.9	9.6	1.28237	.00117	1.25277	.00135
33	160 36.4	8.4	112 43.5	9.8	1.28354	0.00117	1.25142	0.00137
34	160 44.8		112 53.3		1.28471		1.25005	

TABLE II—Continued.

Gr. Sid. Time.		θ	Diff.	θ'	Diff.	log R	Diff.	log R'	Diff.
h	m	o	'	o	'				
7	34	160	44.8	112	53.3	1.28471	+ 0.00117	1.25005	- 0.00138
	35	160	53.2	113	3.2	1.28588	.00116	1.24867	.00139
	36	161	1.6	113	13.2	1.28704	.00116	1.24728	.00141
	37	161	10.0	113	23.4	1.28820	.00116	1.24587	.00142
	38	161	18.4	113	33.7	1.28936	.00115	1.24445	.00143
	39	161	26.8	113	44.1	1.29051	.00115	1.24302	.00144
7	40	161	35.2	113	54.7	1.29166	.00114	1.24158	- 0.00146
	41	161	43.5	114	5.4	1.29280	.00114	1.24012	.00147
	42	161	51.8	114	16.2	1.29394	.00113	1.23865	.00148
	43	162	0.1	114	27.0	1.29507	.00113	1.23717	.00149
	44	162	8.4	114	37.9	1.29620	.00112	1.23568	.00150
	45	162	16.7	114	48.9	1.29732	.00112	1.23418	.00151
	46	162	25.0	115	0.0	1.29844	.00112	1.23267	.00153
	47	162	33.3	115	11.2	1.29956	.00111	1.23114	.00155
	48	162	41.6	115	22.5	1.30067	.00111	1.22959	.00156
	49	162	49.9	115	33.8	1.30178	.00111	1.22803	.00158
7	50	162	58.1	115	45.2	1.30289	+ 0.00111	1.22645	- 0.00160
	51	163	6.3	115	56.8	1.30400	.00110	1.22485	.00160
	52	163	14.5	116	8.6	1.30510	.00110	1.22325	.00161
	53	163	22.7	116	20.6	1.30620	.00110	1.22164	.00164
	54	163	30.9	116	32.7	1.30730	.00109	1.22000	.00165
	55	163	39.1	116	44.9	1.30839	.00109	1.21835	.00166
	56	163	47.3	116	57.2	1.30948	.00108	1.21669	.00168
	57	163	55.5	117	9.6	1.31056	.00108	1.21501	.00169
	58	164	3.7	117	22.2	1.31164	.00107	1.21332	.00171
	59	164	11.9	117	35.0	1.31271	.00107	1.21161	.00172
8	0	164	20.0	117	48.0	1.31378	+ 0.00106	1.20989	- 0.00174
	1	164	28.1	118	1.1	1.31484	.00106	1.20815	.00175
	2	164	36.2	118	14.4	1.31590	.00105	1.20640	.00176
	3	164	44.3	118	27.8	1.31695	.00104	1.20464	.00177
	4	164	52.4	118	41.4	1.31799	.00104	1.20287	.00179
	5	165	0.5	118	55.1	1.31903	.00104	1.20108	.00180
	6	165	8.5	119	8.9	1.32007	.00104	1.19928	.00182
	7	165	16.5	119	22.9	1.32111	.00103	1.19746	.00184
	8	165	24.5	119	37.0	1.32214	.00103	1.19562	.00185
	9	165	32.5	119	51.3	1.32317	.00102	1.19377	.00187
8	10	165	40.5	120	5.8	1.32419	+ 0.00101	1.19190	- 0.00188
	11	165	48.4	120	20.4	1.32520	.00101	1.19002	.00190
	12	165	56.3	120	35.2	1.32621	.00100	1.18812	.00191
	13	166	4.2	120	50.2	1.32721	.00100	1.18621	.00192
	14	166	12.1	121	5.4	1.32821	.00099	1.18429	.00193
	15	166	20.0	121	20.7	1.32920	.00099	1.18236	.00195
	16	166	27.9	121	36.2	1.33019	.00098	1.18041	.00197
	17	166	35.8	121	51.9	1.33117	.00097	1.17844	.00198
	18	166	43.7	122	7.9	1.33214	.00097	1.17646	.00200
	19	166	51.6	122	24.1	1.33311	.00096	1.17446	.00201
8	20	166	59.5	122	40.5	1.33407	+ 0.00095	1.17245	- 0.00202
	21	167	7.3	122	57.0	1.33502		1.17043	

TABLE II—Continued.

Gr. Sid. Time.	θ	Diff.	θ'	Diff.	log R	Diff.	log R'	Diff.
h m	o /	'	o /	'				
8 21	167 7.3		122 57.0		1.33502		1.17043	
22	167 15.1	+ 7.8	123 13.7	+ 16.7	1.33597	+ 0.00095	1.16840	- 0.00203
23	167 22.9	7.8	123 30.6	16.9	1.33691	.00094	1.16636	.00204
24	167 30.7	7.8	123 47.7	17.1	1.33783	.00092	1.16430	.00206
25	167 38.5	7.8	124 5.1	17.4	1.33874	.00091	1.16223	.00207
26	167 46.3	7.8	124 22.6	17.5	1.33965	.00091	1.16015	.00208
27	167 54.1	7.8	124 40.3	17.7	1.34056	.00091	1.15806	.00209
28	168 1.9	7.8	124 58.2	17.9	1.34147	.00090	1.15595	.00211
29	168 9.7	7.8	125 16.3	18.1	1.34237	.00090	1.15382	.00213
8 30	168 17.5	7.8	125 34.6	18.3	1.34327	.00090	1.15168	.00214
31	168 25.2	+ 7.7	125 53.2	+ 18.6	1.34416	+ 0.00089	1.14952	- 0.00216
32	168 32.9	7.7	126 12.0	18.8	1.34504	.00088	1.14735	.00217
33	168 40.6	7.7	126 31.0	19.0	1.34591	.00087	1.14517	.00218
34	168 48.3	7.7	126 50.2	19.2	1.34677	.00086	1.14298	.00219
35	168 56.0	7.7	127 9.6	19.4	1.34762	.00085	1.14078	.00220
36	169 3.6	7.6	127 29.3	19.7	1.34847	.00085	1.13857	.00221
37	169 11.2	7.6	127 49.3	20.0	1.34931	.00084	1.13635	.00222
38	169 18.8	7.6	128 9.5	20.2	1.35014	.00083	1.13411	.00224
39	169 26.4	7.6	128 29.9	20.4	1.35096	.00082	1.13186	.00225
8 40	169 34.0	7.6	128 50.6	20.7	1.35178	.00082	1.12961	.00225
41	169 41.5	+ 7.5	129 11.6	+ 21.0	1.35259	+ 0.00081	1.12735	- 0.00226
42	169 49.0	7.5	129 32.8	21.2	1.35339	.00080	1.12508	.00227
43	169 56.5	7.5	129 54.2	21.4	1.35419	.00080	1.12280	.00228
44	170 4.0	7.5	130 16.0	21.8	1.35498	.00079	1.12050	.00230
45	170 11.5	7.5	130 38.0	22.0	1.35575	.00077	1.11820	.00230
46	170 19.0	7.5	131 0.2	22.2	1.35651	.00076	1.11589	.00231
47	170 26.5	7.5	131 22.7	22.5	1.35726	.00075	1.11358	.00231
48	170 34.0	7.5	131 45.6	22.9	1.35801	.00075	1.11126	.00232
49	170 41.5	7.5	132 8.7	23.1	1.35875	.00074	1.10892	.00234
8 50	170 49.0	7.5	132 32.1	23.4	1.35949	.00074	1.10658	.00234
51	170 56.4	+ 7.4	132 55.8	+ 23.7	1.36022	+ 0.00073	1.10423	- 0.00235
52	171 3.8	7.4	133 19.8	24.0	1.36094	.00072	1.10188	.00235
53	171 11.2	7.4	133 44.0	24.2	1.36164	.00070	1.09952	.00236
54	171 18.5	7.3	134 8.5	24.5	1.36233	.00069	1.09716	.00236
55	171 25.8	7.3	134 33.3	24.8	1.36301	.00068	1.09480	.00236
56	171 33.1	7.3	134 58.4	25.1	1.36368	.00067	1.09244	.00236
57	171 40.4	7.3	135 23.9	25.5	1.36434	.00066	1.09008	.00236
58	171 47.7	7.3	135 49.7	25.8	1.36500	.00066	1.08771	.00237
59	171 55.0	7.3	136 15.8	26.1	1.36566	.00066	1.08534	.00237
9 0	172 2.3	7.3	136 42.1	26.3	1.36631	.00065	1.08297	.00237
1	172 9.5	+ 7.2	137 8.8	+ 26.7	1.36695	+ 0.00064	1.08059	- 0.00238
2	172 16.7	7.2	137 35.8	27.0	1.36759	.00064	1.07821	.00238
3	172 23.9	7.2	138 3.0	27.2	1.36821	.00062	1.07583	.00238
4	172 31.1	7.2	138 30.5	27.5	1.36882	.00061	1.07345	.00238
5	172 38.3	7.2	138 58.3	27.8	1.36942	.00060	1.07108	.00237
6	172 45.5	7.2	139 26.5	28.2	1.37001	.00059	1.06872	.00236
7	172 52.7	7.2	139 55.1	28.6	1.37059	.00058	1.06637	.00235
8	172 59.9	7.2	140 24.0	28.9	1.37117	+ 0.00058	1.06403	- 0.00234

TABLE II—Continued.

Gr. Sid. Time.	θ	Diff.	θ'	Diff.	log R	Diff.	log R'	Diff.
h m	o /	'	o /	'				
9 8	172 59.9		140 24.0		1.37117		1.06403	
9 9	173 7.1	+ 7.2	140 53.3	+ 29.3	1.37174	+ 0.00057	1.06169	- 0.00234
9 10	173 14.3	7.2	141 22.9	29.6	1.37230	.00056	1.05935	.00234
9 11	173 21.4	+ 7.1	141 52.8	+ 29.9	1.37285	+ 0.00055	1.05702	- 0.00233
9 12	173 28.5	7.1	142 23.1	30.3	1.37338	.00053	1.05470	.00232
9 13	173 35.6	7.1	142 53.6	30.5	1.37390	.00052	1.05240	.00230
9 14	173 42.6	7.0	143 24.4	30.8	1.37441	.00051	1.05011	.00229
9 15	173 49.6	7.0	143 55.6	31.2	1.37492	.00051	1.04783	.00228
9 16	173 56.6	7.0	144 27.1	31.5	1.37543	.00051	1.04556	.00227
9 17	174 3.6	7.0	144 59.0	31.9	1.37593	.00050	1.04330	.00226
9 18	174 10.6	7.0	145 31.3	32.3	1.37641	.00048	1.04106	.00224
9 19	174 17.6	7.0	146 3.9	32.6	1.37687	.00046	1.03883	.00223
9 20	174 24.6	+ 7.0	146 36.7	+ 32.8	1.37732	+ 0.00045	1.03661	- 0.00222
9 21	174 31.5	6.9	147 9.9	33.2	1.37776	.00044	1.03441	.00220
9 22	174 38.4	6.9	147 43.5	33.6	1.37820	.00044	1.03224	.00217
9 23	174 45.3	6.9	148 17.4	33.9	1.37864	.00044	1.03010	.00214
9 24	174 52.2	6.9	148 51.5	34.1	1.37907	.00043	1.02799	.00211
9 25	174 59.1	6.9	149 26.0	34.5	1.37950	.00043	1.02590	.00209
9 26	175 6.0	6.9	150 0.9	34.9	1.37992	.00042	1.02383	.00207
9 27	175 12.9	6.9	150 36.2	35.3	1.38032	.00040	1.02177	.00206
9 28	175 19.8	6.9	151 11.7	35.5	1.38071	.00039	1.01974	.00203
9 29	175 26.7	6.9	151 47.5	35.8	1.38108	.00037	1.01773	.00201
9 30	175 33.5	6.8	152 23.7	36.2	1.38143	.00035	1.01575	.00198
9 31	175 40.3	+ 6.8	153 0.1	+ 36.4	1.38178	+ 0.00035	1.01380	- 0.00195
9 32	175 47.1	6.8	153 36.8	36.7	1.38212	.00034	1.01188	.00192
9 33	175 53.9	6.8	154 13.8	37.0	1.38246	.00034	1.00999	.00189
9 34	176 0.6	6.7	154 51.0	37.2	1.38279	.00033	1.00813	.00186
9 35	176 7.3	6.7	155 28.5	37.5	1.38311	.00032	1.00630	.00183
9 36	176 14.0	6.7	156 6.3	37.8	1.38342	.00031	1.00451	.00179
9 37	176 20.7	6.7	156 44.4	38.1	1.38373	.00031	1.00276	.00175
9 38	176 27.4	6.7	157 22.8	38.4	1.38403	.00030	1.00106	.00170
9 39	176 34.1	6.7	158 1.4	38.6	1.38432	.00029	0.99940	.00166
9 40	176 40.8	6.7	158 40.2	38.8	1.38460	.00028	0.99777	.00163
9 41	176 47.4	+ 6.6	159 19.3	+ 39.1	1.38487	+ 0.00027	0.99618	- 0.00159
9 42	176 54.0	6.6	159 58.6	39.3	1.38513	.00026	0.99464	.00154
9 43	177 0.6	6.6	160 38.2	39.6	1.38538	.00025	0.99315	.00149
9 44	177 7.2	6.6	161 18.0	39.8	1.38562	.00024	0.99171	.00144
9 45	177 13.8	6.6	161 58.1	40.1	1.38584	.00022	0.99031	.00140
9 46	177 20.4	6.6	162 38.4	40.3	1.38605	.00021	0.98895	.00136
9 47	177 27.0	6.6	163 19.0	40.6	1.38626	.00021	0.98764	.00131
9 48	177 33.6	6.6	163 59.7	40.7	1.38646	.00020	0.98637	.00127
9 49	177 40.1	6.5	164 40.5	40.8	1.38665	.00019	0.98514	.00123
9 50	177 46.6	6.5	165 21.5	41.0	1.38683	.00018	0.98396	.00118
9 51	177 53.1	+ 6.5	166 2.7	+ 41.2	1.38700	+ 0.00017	0.98284	- 0.00112
9 52	177 59.6	6.5	166 44.0	41.3	1.38717	.00017	0.98178	.00106
9 53	178 6.1	6.5	167 25.4	41.4	1.38733	.00016	0.98077	.00101
9 54	178 12.5	6.4	168 6.9	41.5	1.38748	.00015	0.97981	.00096
9 55	178 18.9	6.4	168 48.5	41.6	1.38762	+ 0.00014	0.97889	- 0.00092

TABLE II—Continued.

Gr. Sid. Time.	θ	Diff.	θ'	Diff.	log R	Diff.	log R'	Diff.
h m	$^{\circ}$ /	'	$^{\circ}$ /	'				
9 55	178 18.9		168 48.5		1.38762		0.97889	
56	178 25.3	+ 6.4	169 30.3	+ 41.8	1.38775	+ 0.00013	0.97803	- 0.00086
57	178 31.7	6.4	170 12.2	41.9	1.38787	.00012	0.97723	.00080
58	178 38.1	6.4	170 54.1	41.9	1.38798	.00011	0.97649	.00074
59	178 44.5	6.4	171 36.1	42.0	1.38808	.00010	0.97580	.00069
10 0	178 50.9	6.4	172 18.1	42.0	1.38817	.00009	0.97516	.00064
1	178 57.2	+ 6.3	173 0.1	+ 42.0	1.38825	+ 0.00008	0.97457	- 0.00059
2	179 3.5	6.3	173 42.2	42.1	1.38832	.00007	0.97403	.00054
3	179 9.8	6.3	174 24.2	42.0	1.38839	.00007	0.97355	.00048
4	179 16.1	6.3	175 6.2	42.0	1.38845	.00006	0.97313	.00042
5	179 22.4	6.3	175 48.2	42.0	1.38850	.00005	0.97276	.00037
6	179 28.7	6.3	176 20.2	42.0	1.38854	.00004	0.97245	.00031
7	179 35.0	6.3	177 2.2	42.0	1.38857	.00003	0.97220	.00025
8	179 41.3	6.3	177 44.2	42.0	1.38860	.00003	0.97201	.00019
9	179 47.6	6.3	178 26.2	42.0	1.38863	.00003	0.97188	.00013
10 10	179 53.9	6.3	179 18.1	41.9	1.38865	.00002	0.97180	.00008
11	180 0.1	+ 6.2	179 59.9	+ 41.8	1.38866	+ 0.00001	0.97177	- 0.00003
12	180 6.3	6.2	180 41.6	41.7	1.38865	- 0.00001	0.97180	+ 0.00003
13	180 12.5	6.2	181 23.1	41.5	1.38864	.00001	0.97188	.00008
14	180 18.6	6.1	182 4.4	41.3	1.38862	.00002	0.97202	.00014
15	180 24.7	6.1	182 45.5	41.1	1.38859	.00003	0.97221	.00019
16	180 30.8	6.1	183 26.5	41.0	1.38856	.00003	0.97245	.00024
17	180 36.9	6.1	184 7.4	40.9	1.38852	.00004	0.97274	.00029
18	180 43.0	6.1	184 48.1	40.7	1.38847	.00005	0.97307	.00033
19	180 49.1	6.1	185 28.7	40.6	1.38841	.00006	0.97344	.00037
10 20	180 55.2	6.1	186 9.1	40.4	1.38834	.00007	0.97386	.00042
21	181 1.2	+ 6.0	186 49.3	+ 40.2	1.38827	- 0.00007	0.97433	+ 0.00047
22	181 7.2	6.0	187 29.4	40.1	1.38819	.00008	0.97886	.00053
23	181 13.2	6.0	188 9.2	39.8	1.38810	.00009	0.97544	.00058
24	181 19.2	6.0	188 48.7	39.5	1.38800	.00010	0.97606	.00062
25	181 25.2	6.0	189 28.1	39.4	1.38789	.00011	0.97673	.00067
26	181 31.2	6.0	190 7.3	39.2	1.38778	.00011	0.97744	.00071
27	181 37.1	5.9	190 46.1	38.8	1.38766	.00012	0.97819	.00075
28	181 43.0	5.9	191 24.7	38.6	1.38754	.00012	0.97899	.00080
29	181 48.9	5.9	192 3.0	38.3	1.38742	.00012	0.97984	.00085
10 30	181 54.8	5.9	192 40.9	37.9	1.38729	.00013	0.98073	.00089
31	182 0.7	+ 5.9	193 18.6	+ 37.7	1.38715	- 0.00014	0.98166	+ 0.00093
32	182 6.6	5.9	193 56.1	37.5	1.38700	.00015	0.98263	.00097
33	182 12.5	5.9	194 33.3	37.2	1.38684	.00016	0.98363	.00100
34	182 18.4	5.9	195 10.3	37.0	1.38667	.00017	0.98466	.00103
35	182 24.3	5.9	195 47.0	36.7	1.38650	.00017	0.98572	.00106
36	182 30.1	5.8	196 23.4	36.4	1.38633	.00017	0.98682	.00110
37	182 35.9	5.8	196 59.4	36.0	1.38615	.00018	0.98795	.00113
38	182 41.7	5.8	197 35.1	35.7	1.38596	.00019	0.98911	.00116
39	182 47.5	5.8	198 10.5	35.4	1.38576	.00020	0.99031	.00120
10 40	182 53.3	5.8	198 45.6	35.1	1.38556	.00020	0.99155	.00124
41	182 59.0	5.7	199 20.4	34.8	1.38536	.00020	0.99282	.00127
42	183 4.7	+ 5.7	199 54.9	+ 34.5	1.38515	- 0.00021	0.99411	+ 0.00129

TABLE II—Continued.

Gr. Sid. Time.	θ	Diff.	θ'	Diff.	log R	Diff.	log R'	Diff.
h m	o ' "	'	o ' "	'				
10 42	183 4.7		199 54.9		1.38515		0.99411	
43	183 10.4	+ 5.7	200 29.0	+ 34.1	1.38493	- 0.00022	0.99542	+ 0.00131
44	183 16.1	5.7	201 2.8	33.8	1.38471	.00022	0.99676	.00134
45	183 21.8	5.7	201 36.2	33.4	1.38448	.00023	0.99813	.00137
46	183 27.5	5.7	202 9.3	33.1	1.38425	.00023	0.99952	.00139
47	183 33.2	5.7	202 42.1	32.8	1.38401	.00024	1.00093	.00141
48	183 38.9	5.7	203 14.6	32.5	1.38376	.00025	1.00236	.00143
49	183 44.6	5.7	203 46.7	32.1	1.38350	.00026	1.00381	.00145
10 50	183 50.2	5.6	204 18.5	31.8	1.38324	.00026	1.00528	.00147
51	183 55.8	+ 5.6	204 50.0	+ 31.5	1.38297	- 0.00027	1.00678	+ 0.00150
52	184 1.4	5.6	205 21.2	31.2	1.38270	.00027	1.00830	.00152
53	184 7.0	5.6	205 52.0	30.8	1.38243	.00027	1.00983	.00153
54	184 12.6	5.6	206 22.5	30.5	1.38215	.00028	1.01137	.00154
55	184 18.1	5.5	206 52.7	30.2	1.38187	.00028	1.01292	.00155
56	184 23.6	5.5	207 22.5	29.8	1.38158	.00029	1.01448	.00156
57	184 29.1	5.5	207 51.9	29.4	1.38129	.00029	1.01606	.00158
58	184 34.6	5.5	208 21.0	29.1	1.38099	.00030	1.01765	.00159
59	184 40.1	5.5	208 49.8	28.8	1.38068	.00031	1.01925	.00160
11 0	184 45.6	5.5	209 18.2	28.4	1.38037	.00031	1.02086	.00161
1	184 51.1	+ 5.5	209 46.4	+ 28.2	1.38005	- 0.00032	1.02249	+ 0.00163
2	184 56.6	5.5	210 14.3	27.9	1.37972	.00033	1.02414	.00165
3	185 2.0	5.4	210 42.0	27.7	1.37939	.00033	1.02580	.00166
4	185 7.4	5.4	211 9.2	27.2	1.37906	.00033	1.02746	.00166
5	185 12.8	5.4	211 35.9	26.7	1.37873	.00033	1.02912	.00166
6	185 18.2	5.4	212 2.4	26.5	1.37840	.00033	1.03079	.00167
7	185 23.6	5.4	212 28.7	26.3	1.37806	.00034	1.03246	.00167
8	185 29.0	5.4	212 54.6	25.9	1.37771	.00035	1.03413	.00167
9	185 34.4	5.4	213 20.2	25.6	1.37736	.00035	1.03581	.00168
11 10	185 39.8	5.4	213 45.6	25.4	1.37701	.00035	1.03749	.00168
11	185 45.1	+ 5.3	214 10.6	+ 25.0	1.37666	- 0.00035	1.03917	+ 0.00168
12	185 50.4	5.3	214 35.3	24.7	1.37630	.00036	1.04086	.00169
13	185 55.7	5.3	214 59.8	24.5	1.37594	.00036	1.04255	.00169
14	186 1.0	5.3	215 23.9	24.1	1.37558	.00036	1.04424	.00169
15	186 6.3	5.3	215 47.7	23.8	1.37521	.00037	1.04593	.00169
16	186 11.6	5.3	216 11.2	23.5	1.37484	.00037	1.04763	.00170
17	186 16.9	5.3	216 34.5	23.3	1.37446	.00038	1.04933	.00170
18	186 22.1	5.2	216 57.5	23.0	1.37408	.00038	1.05103	.00170
19	186 27.3	5.2	217 20.4	22.9	1.37369	.00039	1.05274	.00171
11 20	186 32.5	5.2	217 43.0	22.6	1.37330	.00039	1.05445	.00171
21	186 37.7	+ 5.2	218 5.2	+ 22.2	1.37291	- 0.00039	1.05615	+ 0.00170
22	186 42.9	5.2	218 27.0	21.8	1.37251	.00040	1.05785	.00170
23	186 48.1	5.2	218 48.6	21.6	1.37211	.00040	1.05954	.00169
24	186 53.3	5.2	219 10.0	21.4	1.37171	.00040	1.06122	.00168
25	186 58.4	5.1	219 31.2	21.2	1.37130	.00041	1.06290	.00168
26	187 3.5	5.1	219 52.2	21.0	1.37089	.00041	1.06458	.00168
27	187 8.6	5.1	220 12.9	20.7	1.37048	.00041	1.06625	.00167
28	187 13.7	5.1	220 33.2	20.3	1.37006	.00042	1.06792	.00167
29	187 18.8	+ 5.1	220 53.3	+ 20.1	1.36964	- 0.00042	1.06959	+ 0.00167

TABLE II—Continued.

Gr. Sid. Time.	θ	Diff.	θ'	Diff.	log R	Diff.	log R'	Diff.
h m	° '	'	° '	'				
II 29	187 18.8		220 53.3		1.36964		1.06959	
30	187 23.9	+ 5.1	221 13.2	+ 19.9	1.36922	- 0.00042	1.07125	+ 0.00166
31	187 28.9	+ 5.0	221 32.9	+ 19.7	1.36879	- 0.00043	1.07291	+ 0.00166
32	187 33.9	5.0	221 52.3	19.4	1.36836	.00043	1.07456	.00165
33	187 38.9	5.0	222 11.4	19.1	1.36793	.00043	1.07621	.00165
34	187 43.9	5.0	222 30.3	18.9	1.36750	.00043	1.07785	.00164
35	187 48.9	5.0	222 49.1	18.8	1.36707	.00043	1.07949	.00164
36	187 53.9	5.0	223 7.8	18.7	1.36664	.00043	1.08113	.00164
37	187 58.9	5.0	223 26.2	18.4	1.36620	.00044	1.08276	.00163
38	188 3.9	5.0	223 44.3	18.1	1.36576	.00044	1.08438	.00162
39	188 8.9	5.0	224 2.1	17.8	1.36532	.00044	1.08599	.00161
II 40	188 13.9	5.0	224 19.8	17.7	1.36488	.00044	1.08759	.00160
41	188 18.8	+ 4.9	224 37.2	+ 17.4	1.36443	- 0.00045	1.08918	+ 0.00159
42	188 23.7	4.9	224 54.3	17.1	1.36398	.00045	1.09077	.00159
43	188 28.6	4.9	225 11.3	17.0	1.36353	.00045	1.09235	.00158
44	188 33.5	4.9	225 28.2	16.9	1.36307	.00046	1.09393	.00158
45	188 38.4	4.9	225 44.8	16.6	1.36261	.00046	1.09551	.00158
46	188 43.3	4.9	226 1.3	16.5	1.36215	.00046	1.09708	.00157
47	188 48.2	4.9	226 17.7	16.4	1.36169	.00046	1.09864	.00156
48	188 53.1	4.9	226 33.8	16.1	1.36123	.00046	1.10019	.00155
49	188 58.0	4.9	226 49.7	15.9	1.36077	.00046	1.10172	.00153
II 50	189 2.8	4.8	227 5.3	15.6	1.36030	.00047	1.10324	.00152
51	189 7.6	+ 4.8	227 20.7	+ 15.4	1.35983	- 0.00047	1.10476	+ 0.00152
52	189 12.4	4.8	227 35.9	15.2	1.35935	.00048	1.10627	.00151
53	189 17.2	4.8	227 51.0	15.1	1.35887	.00048	1.10777	.00150
54	189 22.0	4.8	228 6.0	15.0	1.35839	.00048	1.10926	.00149
55	189 26.8	4.8	228 20.8	14.8	1.35791	.00048	1.11074	.00148
56	189 31.5	4.7	228 35.5	14.7	1.35743	.00048	1.11222	.00148
57	189 36.2	4.7	228 50.1	14.6	1.35695	.00048	1.11369	.00147
58	189 40.9	4.7	229 4.4	14.3	1.35647	.00048	1.11515	.00146
59	189 45.6	4.7	229 18.4	14.0	1.35599	.00048	1.11660	.00145
II 12	0 189 50.3	4.7	229 32.3	13.9	1.35551	.00048	1.11804	.00144
1	189 55.0	+ 4.7	229 46.1	+ 13.8	1.35503	- 0.00048	1.11947	+ 0.00143
2	189 59.7	4.7	229 59.7	13.6	1.35455	.00048	1.12089	.00142
3	190 4.4	4.7	230 13.2	13.5	1.35406	.00049	1.12230	.00141
4	190 9.1	4.7	230 26.6	13.4	1.35357	.00049	1.12370	.00140
5	190 13.7	4.6	230 39.9	13.3	1.35308	.00049	1.12510	.00140
6	190 18.3	4.6	230 53.0	13.1	1.35259	.00049	1.12650	.00140
7	190 22.9	4.6	231 6.0	13.0	1.35210	.00049	1.12789	.00139
8	190 27.5	4.6	231 18.8	12.8	1.35161	.00049	1.12927	.00138
9	190 32.1	4.6	231 31.3	12.5	1.35112	.00049	1.13063	.00136
II 12	10 190 36.7	4.6	231 43.7	12.4	1.35063	.00049	1.13198	.00135
11	190 41.3	+ 4.6	231 56.0	+ 12.3	1.35013	- 0.00050	1.13332	+ 0.00134
12	190 45.9	4.6	232 8.1	12.1	1.34963	.00050	1.13465	.00133
13	190 50.4	4.5	232 20.0	11.9	1.34913	.00050	1.13598	.00133
14	190 54.9	4.5	232 31.9	11.9	1.34863	.00050	1.13730	.00132
15	190 59.4	4.5	232 43.7	11.8	1.34813	.00050	1.13861	.00131
16	191 3.9	4.5	232 55.5	11.8	1.34763	.00050	1.13991	.00130

TABLE II—Continued.

Gr. Sid. Time.		θ		Diff.	θ'		Diff.	log R	Diff.	log R'	Diff.
h	m	o	'	'	o	'	'				
12	16	191	3.9		232	55.5		1.34763		1.13991	
	17	191	8.4	+ 4.5	233	7.2	+ 11.7	1.34713	- 0.00050	1.14120	+ 0.00129
	18	191	12.9	4.5	233	18.7	11.5	1.34662	.00051	1.14249	.00129
	19	191	17.4	4.5	233	30.0	11.3	1.34611	.00051	1.14377	.00128
12	20	191	21.9	+ 4.5	233	41.3	+ 11.3	1.34560	- 0.00051	1.14504	+ 0.00127

TABLE III.

Parallactic Elements P, P'.

Gr. Sid. Time.		log P	Diff.	log P'	Diff.	Gr. Sid. Time.	log P	Diff.	log P'	Diff.
h	m					h	m			
6	20	+ 1.27294	- 0.00081	- 1.10183	- 0.00180	6	52	+ 1.24171	- 1.15795	- 0.00171
	21	1.27213	.00082	1.10363	.00179		53	1.24055	1.15966	.00170
	22	1.27131	.00083	1.10542	.00179		54	1.23937	1.16136	.00170
	23	1.27048	.00085	1.10721	.00179		55	1.23817	1.16306	.00170
	24	1.26963	.00086	1.10900	.00178		56	1.23696	1.16476	.00170
	25	1.26877	.00087	1.11078	.00178		57	1.23574	1.16646	.00170
	26	1.26790	.00088	1.11256	.00178		58	1.23450	1.16816	.00170
	27	1.26702	.00089	1.11434	.00178		59	1.23325	1.16985	.00169
	28	1.26613	.00090	1.11612	.00177	7	0	+ 1.23199	- 1.17154	.00169
	29	1.26523	.00090	1.11789	.00177		1	1.23072	1.17323	- 0.00169
6	30	+ 1.26433	- 0.00091	- 1.11966	- 0.00177		2	1.22943	1.17491	.00168
	31	1.26342	.00092	1.12143	.00177		3	1.22812	1.17658	.00167
	32	1.26250	.00093	1.12320	.00176		4	1.22679	1.17825	.00167
	33	1.26157	.00095	1.12496	.00176		5	1.22544	1.17991	.00166
	34	1.26062	.00096	1.12672	.00175		6	1.22408	1.18157	.00166
	35	1.25966	.00097	1.12848	.00175		7	1.22271	1.18323	.00166
	36	1.25869	.00098	1.13023	.00175		8	1.22132	1.18489	.00165
	37	1.25771	.00099	1.13198	.00175		9	1.21992	1.18654	.00165
	38	1.25672	.00099	1.13373	.00174	7	10	+ 1.21850	- 1.18819	.00165
	39	1.25573	.00100	1.13548	.00174		11	1.21707	1.18984	- 0.00165
6	40	+ 1.25473	- 0.00100	- 1.13722	- 0.00174		12	1.21563	1.19148	.00164
	41	1.25373	.00102	1.13896	.00174		13	1.21417	1.19312	.00164
	42	1.25271	.00104	1.14070	.00174		14	1.21269	1.19475	.00163
	43	1.25167	.00105	1.14244	.00173		15	1.21118	1.19637	.00162
	44	1.25062	.00107	1.14418	.00173		16	1.20965	1.19798	.00161
	45	1.24955	.00109	1.14591	.00173		17	1.20810	1.19959	.00161
	46	1.24846	.00110	1.14764	.00172		18	1.20654	1.20119	.00160
	47	1.24736	.00111	1.14937	.00172		19	1.20496	1.20279	.00160
	48	1.24625	.00112	1.15109	.00172	7	20	+ 1.20337	- 1.20439	.00160
	49	1.24513	.00113	1.15281	.00172		21	1.20177	1.20599	- 0.00160
6	50	+ 1.24400	- 0.00114	- 1.15453	- 0.00171		22	1.20015	1.20758	.00159
	51	1.24286	.00115	1.15624	.00171		23	1.19850	1.20917	.00159
	52	+ 1.24171	- 0.00115	- 1.15795	- 0.00171		24	+ 1.19683	- 1.21076	- 0.00159

TABLE III—Continued.

Gr. Sid. Time.	log P	Diff.	log P'	Diff.	Gr. Sid. Time.	log P	Diff.	log P'	Diff.
h m					h m				
7 24	+ 1.19683		— 1.21076		8 11	+ 1.09073		— 1.27796	
25	1.19513	— 0.00170	1.21234	— 0.00158	12	1.08776	— 0.00297	1.27921	— 0.00125
26	1.19341	.00172	1.21391	.00157	13	1.08474	.00302	1.28045	.00124
27	1.19168	.00173	1.21547	.00156	14	1.08166	.00308	1.28169	.00124
28	1.18993	.00175	1.21703	.00156	15	1.07853	.00313	1.28292	.00123
29	1.18817	.00176	1.21858	.00155	16	1.07536	.00317	1.28414	.00122
7 30	+ 1.18638	.00179	— 1.22013	.00155	17	1.07216	.00320	1.28535	.00121
31	1.18456	— 0.00182	1.22167	— 0.00154	18	1.06892	.00324	1.28655	.00120
32	1.18272	.00184	1.22320	.00153	19	1.06565	.00327	1.28775	.00120
33	1.18087	.00185	1.22473	.00153	8 20	+ 1.06234	.00331	— 1.28894	.00119
34	1.17899	.00188	1.22625	.00152	21	1.05898	— 0.00336	1.29012	— 0.00118
35	1.17708	.00191	1.22776	.00151	22	1.05557	.00341	1.29129	.00117
36	1.17515	.00193	1.22927	.00151	23	1.05212	.00345	1.29245	.00116
37	1.17319	.00196	1.23078	.00151	24	1.04862	.00350	1.29360	.00115
38	1.17122	.00197	1.23228	.00150	25	1.04507	.00355	1.29474	.00114
39	1.16923	.00199	1.23378	.00150	26	1.04147	.00360	1.29587	.00113
7 40	+ 1.16722	.00201	— 1.23527	.00149	27	1.03783	.00364	1.29699	.00112
41	1.16519	— 0.00203	1.23675	— 0.00148	28	1.03414	.00369	1.29810	.00111
42	1.16314	.00205	1.23823	.00148	29	1.03040	.00374	1.29920	.00110
43	1.16106	.00208	1.23971	.00148	8 30	+ 1.02661	.00379	— 1.30028	.00108
44	1.15894	.00212	1.24118	.00147	31	1.02276	— 0.00385	1.30135	— 0.00107
45	1.15678	.00216	1.24264	.00146	32	1.01886	.00390	1.30241	.00106
46	1.15459	.00219	1.24409	.00145	33	1.01492	.00394	1.30346	.00105
47	1.15238	.00221	1.24553	.00144	34	1.01092	.00400	1.30450	.00104
48	1.15016	.00222	1.24696	.00143	35	1.00687	.00405	1.30553	.00103
49	1.14792	.00224	1.24839	.00143	36	1.00276	.00411	1.30655	.00102
7 50	+ 1.14565	.00227	— 1.24981	.00142	37	0.99859	.00417	1.30757	.00102
51	1.14335	— 0.00230	1.25123	— 0.00142	38	0.99436	.00423	1.30859	.00102
52	1.14102	.00233	1.25265	.00142	39	0.99007	.00429	1.30960	.00101
53	1.13865	.00237	1.25406	.00141	8 40	+ 0.98572	.00435	— 1.31060	.00100
54	1.13625	.00240	1.25545	.00139	41	0.98130	— 0.00442	1.31159	— 0.00099
55	1.13381	.00244	1.25683	.00138	42	0.97682	.00448	1.31256	.00097
56	1.13135	.00246	1.25820	.00137	43	0.97228	.00454	1.31352	.00096
57	1.12887	.00248	1.25956	.00136	44	0.96767	.00461	1.31447	.00095
58	1.12636	.00251	1.26092	.00136	45	0.96299	.00468	1.31541	.00094
59	1.12382	.00254	1.26227	.00135	46	0.95825	.00474	1.31633	.00092
8 0	+ 1.12126	.00256	— 1.26362	.00135	47	0.95344	.00481	1.31723	.00090
1	1.11868	— 0.00258	1.26497	— 0.00135	48	0.94856	.00488	1.31812	.00089
2	1.11607	.00261	1.26631	.00134	49	0.94361	.00495	1.31901	.00089
3	1.11341	.00266	1.26764	.00133	8 50	+ 0.93858	.00503	— 1.31989	.00088
4	1.11071	.00270	1.26896	.00132	51	0.93347	— 0.00511	1.32077	— 0.00088
5	1.10795	.00276	1.27027	.00131	52	0.92829	.00518	1.32164	.00087
6	1.10515	.00280	1.27157	.00130	53	0.92303	.00526	1.32249	.00085
7	1.10232	.00283	1.27286	.00129	54	0.91768	.00535	1.32332	.00083
8	1.09947	.00285	1.27415	.00129	55	0.91226	.00542	1.32414	.00082
9	1.09659	.00288	1.27543	.00128	56	0.90677	.00549	1.32495	.00081
8 10	+ 1.09367	.00292	— 1.27670	.00127	57	0.90118	.00559	1.32575	.00080
11	+ 1.09073	— 0.00294	— 1.27796	— 0.00126	58	+ 0.89549	— 0.00569	— 1.32654	— 0.00079

TABLE III—Continued

Gr. Sid. Time.	log P	Diff.	log P'	Diff.	Gr. Sid. Time.	log P	Diff.	log P'	Diff.
h m					h m				
8 58	+ 0.89549		- 1.32654		9 45	+ 0.44552		- 1.35122	
59	0.88970	- 0.00579	1.32733	- 0.00079	46	0.42817	- 0.01735	1.35147	- 0.00025
9 0	+ 0.88381	- 0.00589	- 1.32811	- 0.00078	47	0.41019	.01798	1.35171	.00024
1	0.87781	.00600	1.32888	.00077	48	0.39141	.01878	1.35195	.00024
2	0.87172	.00609	1.32964	.00076	49	0.37179	.01962	1.35218	.00023
3	0.86554	.00618	1.33038	.00074	9 50	+ 0.35126	- 0.02053	- 1.35240	- 0.00022
4	0.85926	.00628	1.33110	.00072	51	0.32972	.02154	1.35261	.00021
5	0.85287	.00639	1.33180	.00070	52	0.30708	.02264	1.35281	.00020
6	0.84638	.00649	1.33249	.00069	53	0.28331	.02377	1.35300	.00019
7	0.83978	.00660	1.33318	.00069	54	0.25813	.02518	1.35317	.00017
8	0.83306	.00672	1.33386	.00068	55	0.23143	.02670	1.35333	.00016
9	0.82622	.00684	1.33454	.00068	56	0.20306	.02837	1.35348	.00015
9 10	+ 0.81925	.00697	- 1.33521	.00067	57	0.17270	.03036	1.35362	.00014
11	0.81216	- 0.00709	1.33586	- 0.00065	58	0.14015	.03255	1.35375	.00013
12	0.80493	.00723	1.33650	.00064	59	0.10496	.03519	1.35387	.00012
13	0.79754	.00739	1.33713	.00063	10 0	+ 0.06678	.03818	- 1.35398	.00011
14	0.79002	.00752	1.33774	.00061	1	0.02497	- 0.04181	1.35409	- 0.00011
15	0.78235	.00767	1.33834	.00060	2	9.97881	.04616	1.35419	.00010
16	0.77456	.00779	1.33893	.00059	3	9.92723	.05158	1.35428	.00009
17	0.76662	.00794	1.33951	.00058	4	9.86884	.05839	1.35435	.00007
18	0.75852	.00810	1.34008	.00057	5	9.80145	.06739	1.35441	.00006
19	0.75026	.00826	1.34065	.00057	6	9.72187	.07958	1.35445	.00004
9 20	+ 0.74182	- 0.00844	- 1.34121	- 0.00056	7	9.62462	.09725	1.35449	.00004
21	0.73318	.00864	1.34175	.00054	8	9.49950	.12512	1.35452	.00003
22	0.72435	.00883	1.34227	.00052	9	9.32366	.17584	1.35453	.00001
23	0.71533	.00902	1.34278	.00051	10 10	+ 9.02407	- 0.29959	- 1.35454	- 0.00001
24	0.70612	.00921	1.34328	.00050	11	+ 6.96368	- 2.06039	1.35454	.00000
25	0.69672	.00940	1.34377	.00049	12	- 9.01539	1.35453	+ 0.00001
26	0.68710	.00962	1.34424	.00047	13	9.31774	- 0.30235	1.35452	.00001
27	0.67726	.00984	1.34470	.00046	14	9.49405	.17631	1.35450	.00002
28	0.66719	.01007	1.34516	.00046	15	9.61875	.12470	1.35448	.00002
29	0.65687	.01032	1.34561	.00045	16	9.71526	.09651	1.35445	.00003
9 30	+ 0.64629	.01058	- 1.34605	.00044	17	9.79401	.07875	1.35441	.00004
31	0.63545	- 0.01084	1.34648	- 0.00043	18	9.86046	.06645	1.35435	.00006
32	0.62433	.01112	1.34689	.00041	19	9.91792	.05746	1.35428	.00007
33	0.61289	.01144	1.34729	.00040	10 20	- 9.96856	- 0.05064	- 1.35420	+ 0.00008
34	0.60113	.01176	1.34768	.00039	21	0.01376	.04520	1.35412	.00008
35	0.58906	.01207	1.34806	.00038	22	0.05455	.04079	1.35403	.00009
36	0.57663	.01243	1.34842	.00036	23	0.09177	.03722	1.35393	.00010
37	0.56382	.01281	1.34877	.00035	24	0.12592	.03415	1.35382	.00011
38	0.55063	.01319	1.34911	.00034	25	0.15751	.03159	1.35370	.00012
39	0.53706	.01357	- 1.34945	.00034	26	0.18684	.02933	1.35357	.00013
9 40	+ 0.52308	.01398	1.34978	.00033	27	0.21422	.02738	1.35343	.00014
41	0.50875	- 0.01443	1.35010	- 0.00032	28	0.23991	.02569	1.35329	.00014
42	0.49373	.01502	1.35040	.00030	29	0.26408	.02417	1.35314	.00015
43	0.47822	.01551	1.35069	.00029	10 30	- 0.28687	.02279	- 1.35298	.00016
44	0.46216	.01606	1.35096	.00027	31	0.30848	- 0.02161	1.35281	+ 0.00017
45	+ 0.44552	- 0.01664	- 1.35122	- 0.00026	32	- 0.32899	- 0.02051	- 1.35263	+ 0.00018

TABLE III—Continued.

Gr. Sid. Time.	log P	Diff.	log P'	Diff.	Gr. Sid. Time.	log P	Diff.	log P'	Diff.
h m 10 32	— 0.32899		— 1.35263		h m 11 19	— 0.80022		— 1.33707	
33	0.34850	— 0.01951	1.35245	+ 0.00018	11 20	— 0.80559	— 0.00537	— 1.33662	+ 0.00045
34	0.36712	.01862	1.35226	.00019	21	0.81089	— 0.00530	1.33615	+ 0.00047
35	0.38489	.01777	1.35207	.00019	22	0.81607	.00518	1.33567	.00048
36	0.40192	.01703	1.35188	.00019	23	0.82118	.00511	1.33519	.00048
37	0.41821	.01629	1.35168	.00020	24	0.82624	.00506	1.33471	.00048
38	0.43389	.01568	1.35146	.00022	25	0.83117	.00493	1.33423	.00048
39	0.44893	.01504	1.35123	.00023	26	0.83605	.00488	1.33375	.00048
10 40	— 0.46347	.01454	— 1.35099	.00024	27	0.84082	.00477	1.33327	.00048
41	0.47749	— 0.01402	1.35074	+ 0.00025	28	0.84552	.00470	1.33278	.00049
42	0.49094	.01345	1.35049	.00025	29	0.85018	.00466	1.33229	.00049
43	0.50394	.01300	1.35024	.00025	11 30	— 0.85470	.00452	— 1.33179	.00050
44	0.51649	.01255	1.34998	.00026	31	0.85922	— 0.00450	1.33128	+ 0.00051
45	0.52866	.01217	1.34971	.00027	32	0.86362	.00440	1.33077	.00051
46	0.54053	.01187	1.34943	.00028	33	0.86798	.00436	1.33026	.00051
47	0.55196	.01143	1.34915	.00028	34	0.87223	.00425	1.32975	.00051
48	0.56309	.01113	1.34886	.00029	35	0.87643	.00420	1.32923	.00052
49	0.57383	.01074	1.34856	.00030	36	0.88055	.00412	1.32871	.00052
10 50	— 0.58430	.01047	— 1.34825	.00031	37	0.88463	.00408	1.32818	.00053
51	0.59455	— 0.01025	1.34794	+ 0.00031	38	0.88866	.00403	1.32765	.00053
52	0.60444	.00989	1.34762	.00032	39	0.89261	.00395	1.32712	.00053
53	0.61410	.00966	1.34728	.00034	11 40	— 0.89651	.00390	— 1.32659	.00053
54	0.62346	.00936	1.34694	.00034	41	0.90039	— 0.00388	1.32606	+ 0.00053
55	0.63265	.00919	1.34659	.00035	42	0.90417	.00378	1.32552	.00054
56	0.64159	.00894	1.34624	.00035	43	0.90793	.00376	1.32497	.00055
57	0.65028	.00869	1.34599	.00035	44	0.91160	.00367	1.32442	.00055
58	0.65880	.00852	1.34564	.00035	45	0.91517	.00357	1.32387	.00055
59	0.66711	.00831	1.34528	.00036	46	0.91873	.00356	1.32332	.00055
11 0	— 0.67523	.00812	— 1.34491	.00037	47	0.92226	.00353	1.32277	.00055
1	0.68322	— 0.00799	1.34454	+ 0.00037	48	0.92571	.00345	1.32222	.00055
2	0.69095	.00773	1.34417	.00037	49	0.92913	.00342	1.32167	.00055
3	0.69854	.00759	1.34380	.00037	11 50	— 0.93252	.00339	— 1.32112	.00055
4	0.70592	.00738	1.34342	.00038	51	0.93583	— 0.00331	1.32056	+ 0.00056
5	0.71309	.00717	1.34303	.00039	52	0.93912	.00329	1.32000	.00056
6	0.72013	.00704	1.34263	.00040	53	0.94239	.00327	1.31943	.00057
7	0.72708	.00695	1.34223	.00040	54	0.94558	.00319	1.31886	.00057
8	0.73382	.00674	1.34182	.00041	55	0.94875	.00317	1.31829	.00057
9	0.74047	.00665	1.34141	.00041	56	0.95185	.00310	1.31772	.00057
11 10	— 0.74692	.00645	— 1.34100	.00041	57	0.95492	.00307	1.31715	.00057
11	0.75330	— 0.00638	1.34058	+ 0.00042	58	0.95798	.00306	1.31658	.00057
12	0.75959	.00629	1.34015	.00043	59	0.96097	.00299	1.31601	.00057
13	0.76571	.00612	1.33972	.00043	12 0	— 0.96391	.00294	— 1.31543	.00058
14	0.77173	.00602	1.33929	.00043	1	0.96687	— 0.00296	1.31485	+ 0.00058
15	0.77761	.00588	1.33885	.00044	2	0.96975	.00288	1.31427	.00058
16	0.78341	.00580	1.33841	.00044	3	0.97262	.00287	1.31369	.00058
17	0.78913	.00572	1.33797	.00044	4	0.97540	.00278	1.31310	.00059
18	0.79471	.00558	1.33752	.00045	5	0.97818	.00278	1.31251	.00059
19	— 0.80022	— 0.00551	— 1.33707	+ 0.00045	6	— 0.98090	— 0.00272	— 1.31192	+ 0.00059

TABLE III—Continued.

Gr. Sid. Time.	log P	Diff.	log P'	Diff.	Gr. Sid. Time.	log P	Diff.	log P'	Diff.
h m 12 6	— 0.98090		— 1.31192		h m 12 13	— 0.99912		— 1.30772	
7	0.98359	— .00269	1.31132	+ 0.00060	14	1.00163	— 0.00251	1.30712	+ 0.00060
8	0.98627	.00268	1.31072	.00060	15	1.00408	.00245	1.30652	.00060
9	0.98890	.00263	1.31012	.00060	16	1.00651	.00243	1.30592	.00060
12 10	— 0.99148	.00258	— 1.30952	.00060	17	1.00887	.00236	1.30531	.00061
11	0.99408	— 0.00260	1.30892	+ 0.00060	18	1.01124	.00237	1.30470	.00061
12	0.99662	.00254	1.30832	.00060	19	1.01359	.00235	1.30408	.00062
13	— 0.99912	— 0.00250	— 1.30772	+ 0.00060	12 20	— 1.01585	— 0.00226	— 1.30346	+ 0.00062

§ 10. COMPARISON OF OBSERVED AND TABULAR POSITIONS OF VENUS ON THE FACE OF THE SUN.

The comparison of the observed positions, as deduced from the measures of the photographic plates, with those computed from the theory, are presented in the following tables in a form which admits of their ready translation into equations of condition. There are two sets of results, the one giving position-angles, the other distances.

The second column of each set of tables gives the distances or position-angles derived from observation in the first eight sections of the preceding discussion.

The third column gives the corresponding distances or position-angles derived from the tables in the last two sections. This is followed by the corrections which would result from changes in the longitude of the station, in the relative right ascension and declination of the two bodies, and in the adopted mean solar parallax ($8''.848$). The quantity $\delta\lambda$ is the correction to the provisional west longitude of the station, expressed in seconds of time. The provisional longitudes to be corrected are found on p. 21. The co-efficient of $\delta\lambda$ shows the correction to be applied to the tabular element for each second of time that the station is removed toward the west.

The last column shows the excess of the observed distance or position-angle above that computed, and is the constant term in the equation of condition to be derived from each comparison.

If the observed distances or position-angles needed no further correction, and if $\delta\lambda$ were satisfactorily known, nothing would remain but the easy task of solving the equations thus formed. But the photographic positions of Venus on the Sun's limb may still need several classes of corrections, all arising from the absorption of the solar and terrestrial atmosphere. These corrections are as follows:

First, Mr. J. HOMER LANE, of Washington, called attention to the fact that a vertical displacement of Venus, relative to the center of the Sun, would be caused by the absorption of the solar atmosphere, combined with the chromatic dispersion of the terrestrial atmosphere. His communication is intended to appear in the appendix to this chapter, and may be referred to for a discussion of the action of this cause. At present it will suffice to say that the light emanating from the limb of the Sun has

more red rays than that emanating from the interior of the disc on which Venus is projected. Consequently the rays emanating from the circumference of the disc of Venus will be more refracted by the atmosphere of the earth than those emanating from the Sun's limb. The position of Venus being measured from the limb, the planet will appear more elevated above the horizon than it should appear. The effect of relative parallax being to depress the planet, this cause will operate to apparently diminish the parallax. Thus the parallax will be too small if the effect is not allowed for.

Secondly, the nearer the Sun approaches the horizon, the greater the absorption of the photographic rays by the Earth's atmosphere. Hence, in each photograph, the intensity of the rays from the upper limb of the Sun must be greater than that of the rays from the lower limb. The upper limb will therefore be more enlarged by photographic irradiation than will the lower one, so that the Sun will appear too high on the plate. Venus will therefore be too low relative to the center of the Sun's apparent disc, and the parallax will come out too great from this cause. This last effect, it will be seen, is the opposite of the first.

Thirdly, the photographic intensity of the Sun's rays diminishes from the center toward the circumference of the disc. The planet will, when near the Sun's limb, be less intensely photographed on its outer limb than on its inner limb. It will therefore be apparently drawn toward the Sun's limb by photographic irradiation. The observed distances must therefore be too great from this cause.

The determination of the numerical value of each of these corrections requires a special investigation, which has not yet been undertaken. Without such investigation, it is not possible to say whether the causes just described will appreciably affect the solar parallax. The determination of the numerical value of the first effect is not difficult. It is only necessary to form separate curves of the photographic intensity of different parts of the spectrum for the limb of the Sun and for its center, and to combine this with atmospheric dispersion. The second cause may be determined by finding the increased amount of photographic irradiation due to increased intensity of the impression on the plate. As a general rule, the exposure was so considerable that there is no striking difference of intensity between the limb and the center of the Sun on the photographic plates. For this reason I incline to the opinion that the effect in question will not be considerable. The third effect can be deduced from the equations of condition and also from the same data which gave the second effect.

It is to be remarked that all the effects thus described will also be found in any optical determination of the position of Venus on the disc of the Sun, the eye taking the place of the photographic plate and being affected in the same way. It is probable, however, that the ocular effect is smaller than the photographic one.

Some interest would, no doubt, attach to the solution of these equations disregarding the small corrections in question. In accordance, however, with the recommendation of the *Astronomische Gesellschaft*, such a solution has been postponed until other data are ready to be combined with them. As now presented, they are open to investigation and criticism by all astronomers.

The discussion of the errors and discrepancies among the photographic results properly belongs to their final discussion, and is therefore postponed for the present. It will, however, be remarked that the probable error, as indicated by a comparison of different photographs, far exceeds what was originally estimated.

The errors of the position-angles will be found roughly by dividing the residuals by 4, which will give nearly seconds of arc on a great circle. Such an inspection shows that the probable error of each individual photograph is fully one second of arc in each co-ordinate, but that it differs at the different stations. I am of opinion that the principal source of the error is to be found in the undulations of the Sun's limb produced by the atmosphere. These undulations will produce a greater effect in photographs than in visual observations, because in the latter the observer can generally select moments of good seeing to make his observations, whereas the photograph pictures the Sun as it appears at the moment.

It does not, however, seem probable that the method is attended with any considerable systematic error. It will, therefore, be well adapted to give the error of the tabular position of Venus on the face of the Sun. It does not seem likely that the final probable error of this position, as deduced from these photographs, will much exceed $0''.10$, a degree of accuracy which can hardly be exceeded by other methods of observation.

COMPARISON OF OBSERVED AND TABULAR DISTANCES.

WLADIWOSTOK.

No. of Photo.	Observed Distance.	Tabular Distance.					O. — C.
	"	"	"				"
6	870.38	868.31	- 0.026 $\delta \lambda_1$	+ 0.55 δA	+ 0.80 δD	- 1.96 $\delta \pi$	+ 2.07
7	867.36	865.44	- 0.026	+ 0.54	+ 0.81	- 1.98	+ 1.92
13	813.82	812.31	- 0.009	+ 0.34	+ 0.93	- 2.45	+ 1.51
14	813.21	811.33	- 0.008	+ 0.33	+ 0.93	- 2.46	+ 1.88
15	812.11	810.46	- 0.007	+ 0.33	+ 0.94	- 2.47	+ 1.65
31	878.99	876.08	+ 0.028	- 0.14	+ 0.99	- 2.17	+ 2.91
32	882.10	879.20	+ 0.028	- 0.14	+ 0.99	- 2.13	+ 2.90
33	883.26	882.15	+ 0.028	- 0.15	+ 0.99	- 2.14	+ 1.11
34	888.13	885.16	+ 0.028	- 0.16	+ 0.98	- 2.11	+ 2.97
35	889.82	888.14	+ 0.029	- 0.16	+ 0.98	- 2.09	+ 1.68
36	893.13	891.33	+ 0.029	- 0.17	+ 0.98	- 2.08	+ 1.80
37	895.77	894.16	+ 0.030	- 0.18	+ 0.98	- 2.06	+ 1.61
38	898.17	897.54	+ 0.030	- 0.18	+ 0.98	- 2.04	+ 0.63

NAGASAKI.

No. of Photo.	Observed Distance.	Tabular Distance.					O. — C.
"	"	"	"	"	"	"	"
25	943.60	938.35	— 0.036 $\delta \lambda_2$	+ 0.65 δA	+ 0.71 δD	— 1.18 $\delta \pi$	+ 5.25
26	938.56	937.24	— 0.035	+ 0.65	+ 0.71	— 1.19	+ 1.32
28	937.79	935.31	— 0.035	+ 0.65	+ 0.71	— 1.21	+ 2.48
30	935.89	932.63	— 0.035	+ 0.64	+ 0.71	— 1.22	+ 3.26
31	934.89	931.51	— 0.035	+ 0.64	+ 0.72	— 1.23	+ 3.38
33	929.77	927.30	— 0.034	+ 0.64	+ 0.72	— 1.26	+ 2.47
34	928.50	925.92	— 0.034	+ 0.64	+ 0.72	— 1.27	+ 2.58
35	924.40	921.61	— 0.033	+ 0.63	+ 0.73	— 1.31	+ 2.79
36	922.25	920.27	— 0.033	+ 0.63	+ 0.73	— 1.31	+ 1.98
37	922.30	919.25	— 0.033	+ 0.63	+ 0.73	— 1.32	+ 3.05
38	918.33	915.50	— 0.033	+ 0.62	+ 0.74	— 1.35	+ 2.83
44	907.41	904.05	— 0.032	+ 0.61	+ 0.75	— 1.43	+ 3.36
45	903.28	901.18	— 0.031	+ 0.60	+ 0.76	— 1.46	+ 2.10
46	901.33	899.99	— 0.031	+ 0.60	+ 0.76	— 1.47	+ 1.34
48	898.08	897.15	— 0.030	+ 0.59	+ 0.76	— 1.49	+ 0.93
49	897.05	894.57	— 0.030	+ 0.59	+ 0.77	— 1.51	+ 2.48
50	895.65	893.35	— 0.029	+ 0.59	+ 0.77	— 1.52	+ 2.30
51	890.88	888.51	— 0.029	+ 0.58	+ 0.78	— 1.56	+ 2.37
52	887.56	885.44	— 0.029	+ 0.57	+ 0.78	— 1.58	+ 2.12
53	886.66	882.88	— 0.028	+ 0.57	+ 0.78	— 1.60	+ 3.78
54	883.55	881.24	— 0.028	+ 0.57	+ 0.79	— 1.61	+ 2.31
55	881.37	879.28	— 0.027	+ 0.56	+ 0.79	— 1.63	+ 2.09
57	872.90	871.14	— 0.026	+ 0.55	+ 0.80	— 1.68	+ 1.76
58	869.33	868.40	— 0.026	+ 0.54	+ 0.81	— 1.72	+ 0.93
59	869.19	866.09	— 0.025	+ 0.54	+ 0.81	— 1.74	+ 3.10
61	866.76	861.71	— 0.024	+ 0.53	+ 0.82	— 1.77	+ 5.05
62	861.56	860.68	— 0.024	+ 0.52	+ 0.82	— 1.78	+ 0.88
63	862.48	859.39	— 0.024	+ 0.52	+ 0.82	— 1.79	+ 3.09
64	861.43	858.44	— 0.023	+ 0.52	+ 0.83	— 1.80	+ 2.99
65	858.36	853.62	— 0.022	+ 0.50	+ 0.84	— 1.84	+ 4.74
67	846.96	844.70	— 0.021	+ 0.48	+ 0.85	— 1.92	+ 2.26
72	. . .	825.14	— 0.013	+ 0.41	+ 0.90	— 2.11	
74	822.61	820.79	— 0.012	+ 0.38	+ 0.91	— 2.14	+ 1.82
75	823.64	819.73	— 0.010	+ 0.38	+ 0.91	— 2.16	+ 3.91
76	817.57	815.47	— 0.009	+ 0.35	+ 0.92	— 2.21	+ 2.10
77	816.99	814.48	— 0.009	+ 0.34	+ 0.93	— 2.21	+ 2.51
78	817.36	814.21	— 0.009	+ 0.34	+ 0.93	— 2.22	+ 3.15
79	815.41	813.81	— 0.008	+ 0.34	+ 0.93	— 2.22	+ 1.60
80	816.24	813.53	— 0.008	+ 0.33	+ 0.93	— 2.23	+ 2.71
81	814.94	813.27	— 0.008	+ 0.33	+ 0.93	— 2.23	+ 1.67
84	812.65	809.54	— 0.006	+ 0.29	+ 0.95	— 2.27	+ 3.11
93	821.46	819.69	+ 0.012	+ 0.08	+ 1.00	— 2.27	+ 1.77
94	820.56	820.05	+ 0.012	+ 0.07	+ 1.00	— 2.26	+ 0.51
95	821.71	820.56	+ 0.013	+ 0.07	+ 1.00	— 2.26	+ 1.15
96	820.90	820.90	+ 0.013	+ 0.07	+ 1.00	— 2.26	0.00
99	826.99	825.30	+ 0.014	+ 0.04	+ 1.00	— 2.24	+ 1.69

PEKING.

No. of Photo.	Observed Distance.	Tabular Distance.					O.—C.
"	"	"	"	"	"	"	"
15	943.51	940.77	— 0.035 $\delta \lambda_3$	+ 0.65 δA	+ 0.70 δD	— 0.96 $\delta \pi$	+ 2.74
19	928.70	926.68	— 0.034	+ 0.64	+ 0.72	— 1.07	+ 2.02
21	922.20	920.69	— 0.033	+ 0.63	+ 0.73	— 1.11	+ 1.51
22	918.26	917.54	— 0.033	+ 0.63	+ 0.73	— 1.14	+ 0.72
44	875.96	874.21	+ 0.027	— 0.13	+ 0.99	— 2.27	+ 1.75
45	877.36	875.96	+ 0.028	— 0.13	+ 0.99	— 2.26	+ 1.40
46	880.67	877.78	+ 0.028	— 0.14	+ 0.99	— 2.25	+ 2.89
49	885.10	884.67	+ 0.029	— 0.15	+ 0.99	— 2.22	+ 0.43
50	889.51	886.82	+ 0.029	— 0.16	+ 0.98	— 2.22	+ 2.69
51	891.84	888.92	+ 0.030	— 0.16	+ 0.98	— 2.20	+ 2.92
53	896.55	893.59	+ 0.030	— 0.17	+ 0.98	— 2.18	+ 2.96
54	896.08	895.81	+ 0.030	— 0.18	+ 0.98	— 2.17	+ 0.27
56	901.79	900.26	+ 0.031	— 0.19	+ 0.98	— 2.15	+ 1.53
57	904.57	902.67	+ 0.031	— 0.19	+ 0.98	— 2.14	+ 1.90
58	906.09	905.10	+ 0.032	— 0.20	+ 0.98	— 2.13	+ 0.99
59	908.19	907.15	+ 0.032	— 0.20	+ 0.98	— 2.12	+ 1.04
60	911.03	909.38	+ 0.032	— 0.21	+ 0.97	— 2.11	+ 1.65
61	914.48	911.77	+ 0.033	— 0.21	+ 0.97	— 2.09	+ 2.71
63	919.18	916.52	+ 0.033	— 0.22	+ 0.97	— 2.07	+ 2.66
65	922.32	920.88	+ 0.034	— 0.23	+ 0.97	— 2.05	+ 1.44
67	929.62	925.87	+ 0.034	— 0.24	+ 0.97	— 2.03	+ 3.75
68	929.07	927.95	+ 0.034	— 0.24	+ 0.97	— 2.02	+ 1.12
69	933.01	930.93	+ 0.035	— 0.24	+ 0.96	— 2.01	+ 2.08
70	934.16	933.12	+ 0.035	— 0.25	+ 0.96	— 1.99	+ 1.04
71	938.82	935.23	+ 0.035	— 0.25	+ 0.96	— 1.98	+ 3.59
72	941.50	937.40	+ 0.035	— 0.25	+ 0.96	— 1.97	+ 4.10

KERGUELEN.

No. of Photo.	Observed Distance.	Tabular Distance.					O.—C.
"	"	"	"	"	"	"	"
7	916.11	914.17	— 0.027 $\delta \lambda_4$	+ 0.55 δA	+ 0.80 δD	+ 2.31 $\delta \pi$	+ 1.94
17	864.08	863.02	+ 0.015	+ 0.03	+ 1.00	+ 1.40	+ 1.06
18	870.24	866.43	+ 0.016	+ 0.02	+ 1.00	+ 1.37	+ 3.81
19	870.33	867.57	+ 0.016	+ 0.01	+ 1.00	+ 1.36	+ 2.76
20	869.82	869.23	+ 0.016	0.00	+ 1.00	+ 1.35	+ 0.59
25	904.54	901.73	+ 0.025	— 0.10	+ 0.99	+ 1.21	+ 2.81
32	930.83	930.53	+ 0.029	— 0.17	+ 0.98	+ 1.14	+ 0.30
33	987.36	988.65	+ 0.036	— 0.27	+ 0.95	+ 1.07	— 1.29

NOTE.—The Kerguelon distances have been reduced on the supposition that the measures with the rod were from the end and not from the notch. Had the notch length given on pages 72–73 been used, the distances would have been about 4" greater.

HOBART TOWN.

No. of Photo.	Observed Distance.	Tabular Distance.					O. — C.
	"	"	"	"	"	"	"
9	856.94	854.64	+ 0.015 $\delta \lambda_6$	+ 0.04 δA	+ 1.00 δD	+ 1.09 $\delta \pi$	+ 2.30
10	857.55	855.15	+ 0.015	+ 0.04	+ 1.00	+ 1.09	+ 2.40
11	856.84	855.74	+ 0.015	+ 0.03	+ 1.00	+ 1.10	+ 1.10
12	856.54	856.17	+ 0.016	+ 0.03	+ 1.00	+ 1.10	+ 0.37
13	858.79	857.46	+ 0.016	+ 0.02	+ 1.00	+ 1.11	+ 1.33
14	861.65	859.15	+ 0.016	+ 0.02	+ 1.00	+ 1.13	+ 2.50
15	860.29	859.92	+ 0.017	+ 0.01	+ 1.00	+ 1.13	+ 0.37
16	863.73	861.54	+ 0.017	+ 0.01	+ 1.00	+ 1.15	+ 2.19
17	862.94	861.84	+ 0.017	+ 0.01	+ 1.00	+ 1.15	+ 1.10
18	866.06	863.32	+ 0.018	0.00	+ 1.00	+ 1.16	+ 2.74
19	865.50	863.86	+ 0.018	0.00	+ 1.00	+ 1.17	+ 1.64
22	871.82	870.28	+ 0.019	- 0.03	+ 1.00	+ 1.22	+ 1.54
23	871.42	871.14	+ 0.020	- 0.03	+ 1.00	+ 1.22	+ 0.28
24	873.43	871.94	+ 0.020	- 0.03	+ 1.00	+ 1.23	+ 1.49
25	875.15	872.83	+ 0.020	- 0.04	+ 1.00	+ 1.24	+ 2.32
26	876.62	873.80	+ 0.021	- 0.04	+ 1.00	+ 1.24	+ 2.82
27	875.56	875.92	+ 0.021	- 0.05	+ 1.00	+ 1.26	- 0.36
28	877.29	877.05	+ 0.021	- 0.05	+ 1.00	+ 1.27	+ 0.24
29	879.68	878.29	+ 0.021	- 0.05	+ 1.00	+ 1.28	+ 1.39
30	881.13	879.02	+ 0.022	- 0.06	+ 1.00	+ 1.28	+ 2.11
32	884.66	881.24	+ 0.022	- 0.06	+ 1.00	+ 1.30	+ 3.42
33	882.04	881.65	+ 0.023	- 0.06	+ 1.00	+ 1.31	+ 0.39
34	883.36	882.58	+ 0.023	- 0.07	+ 1.00	+ 1.31	+ 0.78
35	885.31	883.44	+ 0.023	- 0.07	+ 1.00	+ 1.31	+ 1.87
36	885.89	884.63	+ 0.023	- 0.07	+ 1.00	+ 1.32	+ 1.26
37	886.69	885.16	+ 0.023	- 0.07	+ 1.00	+ 1.32	+ 1.53
38	888.64	886.63	+ 0.023	- 0.08	+ 1.00	+ 1.33	+ 2.01
39	890.67	888.00	+ 0.023	- 0.08	+ 1.00	+ 1.34	+ 2.67
40	888.89	889.00	+ 0.023	- 0.09	+ 1.00	+ 1.35	- 0.11
41	890.55	891.04	+ 0.024	- 0.09	+ 0.99	+ 1.36	- 0.49
42	894.54	892.15	+ 0.024	- 0.09	+ 0.99	+ 1.37	+ 2.39
43	893.24	893.15	+ 0.025	- 0.10	+ 0.99	+ 1.37	+ 0.09
44	897.43	894.36	+ 0.025	- 0.10	+ 0.99	+ 1.38	+ 3.07
45	895.41	895.34	+ 0.025	- 0.10	+ 0.99	+ 1.39	+ 0.07
46	898.76	896.81	+ 0.025	- 0.10	+ 0.99	+ 1.40	+ 1.95
47	900.05	898.26	+ 0.025	- 0.11	+ 0.99	+ 1.41	+ 1.79
48	900.74	899.81	+ 0.026	- 0.11	+ 0.99	+ 1.42	+ 0.93

CAMPBELLTOWN.

No. of Photo.	Observed Distance.	Tabular Distance.					O. — C.
	"	"	"	"	"	"	"
10	842.78	836.51	+ 0.006 $\delta \lambda_6$	+ 0.17 δA	+ 0.98 δD	+ 0.81 $\delta \pi$	+ 6.27
11	842.08	837.61	+ 0.006	+ 0.16	+ 0.98	+ 0.83	+ 4.47
12	842.94	837.81	+ 0.006	+ 0.15	+ 0.99	+ 0.83	+ 5.13
17	846.14	842.01	+ 0.009	+ 0.11	+ 0.99	+ 0.91	+ 4.13
18	844.18	842.56	+ 0.009	+ 0.11	+ 0.99	+ 0.91	+ 1.62
19	844.88	843.05	+ 0.010	+ 0.10	+ 0.99	+ 0.92	+ 1.83
23	849.98	847.40	+ 0.013	+ 0.08	+ 1.00	+ 0.97	+ 2.58
24	849.93	848.06	+ 0.013	+ 0.07	+ 1.00	+ 0.98	+ 1.87
28	867.24	862.75	+ 0.017	0.00	+ 1.00	+ 1.12	+ 4.49
29	869.98	867.00	+ 0.019	- 0.02	+ 1.00	+ 1.16	+ 2.98
31	877.62	873.06	+ 0.021	- 0.04	+ 1.00	+ 1.21	+ 4.56
32	877.93	874.65	+ 0.021	- 0.04	+ 1.00	+ 1.22	+ 3.28
33	880.67	877.03	+ 0.021	- 0.05	+ 1.00	+ 1.24	+ 3.64
34	882.82	879.77	+ 0.022	- 0.06	+ 1.00	+ 1.25	+ 3.05
35	887.06	883.40	+ 0.022	- 0.07	+ 1.00	+ 1.28	+ 3.66
36	887.10	886.22	+ 0.024	- 0.08	+ 1.00	+ 1.30	+ 0.88
37	891.15	887.83	+ 0.024	- 0.08	+ 1.00	+ 1.31	+ 3.32
38	891.66	889.01	+ 0.024	- 0.09	+ 1.00	+ 1.31	+ 2.65
39	892.87	891.57	+ 0.024	- 0.09	+ 0.99	+ 1.33	+ 1.30
40	898.15	893.52	+ 0.025	- 0.10	+ 0.99	+ 1.34	+ 4.63
41	897.67	895.02	+ 0.025	- 0.10	+ 0.99	+ 1.35	+ 2.65
42	899.78	896.84	+ 0.025	- 0.11	+ 0.99	+ 1.37	+ 2.94
44	904.12	900.40	+ 0.026	- 0.12	+ 0.99	+ 1.39	+ 3.72
45	905.65	902.73	+ 0.026	- 0.12	+ 0.99	+ 1.40	+ 2.92
46	908.42	904.77	+ 0.027	- 0.13	+ 0.99	+ 1.41	+ 3.65
47	909.71	907.50	+ 0.027	- 0.13	+ 0.99	+ 1.43	+ 2.21
48	947.52	941.70	+ 0.032	- 0.20	+ 0.97	+ 1.61	+ 5.82
49	950.80	945.88	+ 0.033	- 0.21	+ 0.97	+ 1.63	+ 4.92
50	952.16	947.93	+ 0.033	- 0.22	+ 0.97	+ 1.64	+ 4.23
51	953.14	950.13	+ 0.033	- 0.22	+ 0.97	+ 1.65	+ 3.01
52	954.54	951.80	+ 0.033	- 0.22	+ 0.97	+ 1.66	+ 2.74
53	956.85	953.95	+ 0.033	- 0.23	+ 0.97	+ 1.67	+ 2.90

QUEENSTOWN.

No. of Photo.	Observed Distance.	Tabular Distance.					O. — C.
	"	"	"	"	"	"	"
114	932.56	930.90	— 0.030 $\delta \lambda_7$	+ 0.60 δA	+ 0.76 δD	+ 0.27 $\delta \pi$	+ 1.66
115	929.99	927.47	— 0.030	+ 0.60	+ 0.76	+ 0.27	+ 2.52
116	926.09	924.39	— 0.030	+ 0.59	+ 0.77	+ 0.27	+ 1.70
117	925.61	922.42	— 0.029	+ 0.59	+ 0.77	+ 0.28	+ 3.19
118	924.75	918.86	— 0.029	+ 0.58	+ 0.77	+ 0.28	+ 5.89
119	918.99	916.39	— 0.029	+ 0.58	+ 0.78	+ 0.28	+ 2.60
120	914.46	911.87	— 0.028	+ 0.57	+ 0.78	+ 0.29	+ 2.59
122	905.53	903.99	— 0.027	+ 0.56	+ 0.80	+ 0.31	+ 1.54
123	905.29	900.76	— 0.026	+ 0.55	+ 0.80	+ 0.31	+ 4.53
124	898.77	897.23	— 0.026	+ 0.54	+ 0.81	+ 0.32	+ 1.54
125	897.02	894.07	— 0.025	+ 0.53	+ 0.81	+ 0.33	+ 2.95
126	891.85	890.00	— 0.024	+ 0.53	+ 0.82	+ 0.34	+ 1.85
127	887.48	886.63	— 0.024	+ 0.52	+ 0.83	+ 0.35	+ 0.85
128	887.68	883.82	— 0.023	+ 0.51	+ 0.83	+ 0.36	+ 3.86
129	884.41	880.92	— 0.022	+ 0.50	+ 0.84	+ 0.37	+ 3.49
130	878.24	876.76	— 0.021	+ 0.49	+ 0.84	+ 0.39	+ 1.48
131	874.71	874.87	— 0.021	+ 0.49	+ 0.85	+ 0.40	— 0.16
132	876.19	872.66	— 0.020	+ 0.48	+ 0.85	+ 0.41	+ 3.53
133	874.07	870.86	— 0.020	+ 0.47	+ 0.86	+ 0.41	+ 3.21
134	870.82	868.84	— 0.019	+ 0.47	+ 0.86	+ 0.42	+ 1.98
135	868.01	866.92	— 0.019	+ 0.46	+ 0.86	+ 0.43	+ 1.09
139 ^(?)	847.92	843.75	— 0.009	+ 0.35	+ 0.92	+ 0.63	+ 4.17
142	846.33	844.47	— 0.009	+ 0.36	+ 0.92	+ 0.62	+ 1.86
143	845.92	842.63	— 0.009	+ 0.34	+ 0.93	+ 0.65	+ 3.29
144	845.63	843.18	— 0.009	+ 0.35	+ 0.93	+ 0.64	+ 2.45
145	843.99	841.74	— 0.008	+ 0.34	+ 0.93	+ 0.67	+ 2.25
151	846.90	845.25	+ 0.010	+ 0.10	+ 0.99	+ 1.20	+ 1.65
153	848.39	845.44	+ 0.011	+ 0.10	+ 0.99	+ 1.21	+ 2.95
154	847.74	845.99	+ 0.011	+ 0.10	+ 0.99	+ 1.22	+ 1.75
155	848.94	847.01	+ 0.012	+ 0.09	+ 0.99	+ 1.24	+ 1.93
156	850.52	849.37	+ 0.013	+ 0.08	+ 1.00	+ 1.27	+ 1.15
158	854.84	851.32	+ 0.013	+ 0.06	+ 1.00	+ 1.30	+ 3.52
159	854.35	852.50	+ 0.014	+ 0.06	+ 1.00	+ 1.32	+ 1.85
160	853.67	854.16	+ 0.014	+ 0.05	+ 1.00	+ 1.34	— 0.49
161	854.98	854.74	+ 0.014	+ 0.05	+ 1.00	+ 1.35	+ 0.24
163	869.50	864.06	+ 0.018	+ 0.00	+ 1.00	+ 1.46	+ 5.44
164	866.46	865.81	+ 0.018	— 0.00	+ 1.00	+ 1.47	+ 0.65
165 ^(?)	868.09	866.72	+ 0.018	— 0.01	+ 1.00	+ 1.48	+ 1.37
166	869.49	868.19	+ 0.018	— 0.01	+ 1.00	+ 1.50	+ 1.30
167	874.04	869.63	+ 0.019	— 0.02	+ 1.00	+ 1.51	+ 4.41
171	879.87	878.29	+ 0.022	— 0.05	+ 1.00	+ 1.59	+ 1.58
172	885.38	882.36	+ 0.022	— 0.06	+ 1.00	+ 1.63	+ 3.02
173	885.78	884.68	+ 0.022	— 0.07	+ 1.00	+ 1.65	+ 1.10
176	916.59	912.18	+ 0.028	— 0.14	+ 1.00	+ 1.84	+ 4.41
177	915.92	913.03	+ 0.028	— 0.14	+ 1.00	+ 1.85	+ 2.89

CHATHAM ISLAND.

No. of Photo.	Observed Distance.	Tabular Distance.					O. — C.
	"	"	"				"
15	864.13	864.34	— 0.018 $\delta \lambda_3$	+ 0.45 δA	+ 0.87 δD	+ 0.33 $\delta \pi$	— 0.21
16	864.96	863.44	— 0.017	+ 0.45	+ 0.87	+ 0.34	+ 1.52
17	864.96	862.36	— 0.017	+ 0.45	+ 0.87	+ 0.35	+ 2.60
24	838.48	836.66	— 0.001	+ 0.26	+ 0.96	+ 0.86	+ 1.82
25	839.01	836.57	— 0.001	+ 0.25	+ 0.96	+ 0.88	+ 2.44
27	839.32	836.84	+ 0.002	+ 0.21	+ 0.97	+ 0.99	+ 2.48
29	842.46	840.64	+ 0.006	+ 0.15	+ 0.99	+ 1.17	+ 1.82

COMPARISON OF OBSERVED AND TABULAR POSITION-ANGLES.

WLADIWOSTOK.

No. of Photo.	Observed Position-Angle.	Tabular Position-Angle.						O. — C.
		°	'	'			'	
6	+ 36 24.7	+ 36	23.0	— 0.257 $\delta \lambda_1$	+ 177 δA	— 141 δD	+ 6.15 $\delta \pi$	+ 1.7
7	35 50.2	35	54.4	— 0.257	+ 178	— 140	+ 6.07	— 4.2
8	35	25.6	— 0.257	+ 180	— 139	+ 5.97	. . .
13	21 47.3	21	49.9	— 0.294	+ 218	— 94	+ 2.53	— 2.6
14	21 20.1	21	15.7	— 0.294	+ 218	— 92	+ 2.37	+ 4.4
15	+ 20 55.4	+ 20	43.6	— 0.293	+ 220	— 90	+ 2.22	+ 11.8
31	— 8 25.4	— 8	30.5	— 0.251	+ 215	+ 35	— 5.88	+ 5.1
32	— 8 51.7	— 8	58.9	— 0.249	+ 213	+ 36	— 5.97	+ 7.2
33	— 9 27.3	— 9	24.8	— 0.247	+ 213	+ 38	— 6.05	— 2.5
34	— 9 47.3	— 9	50.9	— 0.244	+ 211	+ 40	— 6.15	+ 3.6
35	— 10 18.3	— 10	15.8	— 0.243	+ 211	+ 41	— 6.22	— 2.5
36	— 10 39.3	— 10	42.1	— 0.241	+ 209	+ 43	— 6.32	+ 2.8
37	— 11 9.2	— 11	4.8	— 0.241	+ 209	+ 44	— 6.38	— 4.4
38	— 11 25.0	— 11	31.5	— 0.241	+ 207	+ 46	— 6.47	+ 6.5

NAGASAKI.

No. of Photo.	Observed Position-Angle.		Tabular Position-Angle.					O. — C.					
	o	i	o	i	i	δA	δD		$\delta \pi$				
25	+ 45	6.3	+ 45	0.5	- 0.220	$\delta \lambda_2$	+ 143	δA	- 156	δD	+ 7.22	$\delta \pi$	+ 5.8
26	45	5.8	44	53.6	- 0.220		+ 144		- 155		+ 7.22		+ 12.2
28	44	53.4	44	41.6	- 0.222		+ 145		- 155		+ 7.20		+ 11.8
30	44	33.0	44	24.5	- 0.223		+ 146		- 155		+ 7.15		+ 8.5
31	44	25.9	44	17.4	- 0.223		+ 146		- 155		+ 7.13		+ 8.5
33	43	47.5	43	50.2	- 0.225		+ 148		- 154		+ 7.08		- 2.7
34	43	38.1	43	41.0	- 0.226		+ 149		- 154		+ 7.07		- 2.9
35	43	8.7	43	12.2	- 0.227		+ 150		- 153		+ 7.02		- 3.5
36	43	4.9	43	3.1	- 0.229		+ 151		- 153		+ 7.00		+ 1.8
37	42	53.6	42	56.0	- 0.230		+ 151		- 153		+ 6.97		- 2.4
38	42	33.4	42	30.0	- 0.231		+ 153		- 152		+ 6.92		+ 3.4
44	41	8.2	41	6.7	- 0.238		+ 158		- 150		+ 6.72		+ 1.5
45	40	47.7	40	44.8	- 0.238		+ 160		- 149		+ 6.67		+ 2.9
46	40	30.6	40	35.7	- 0.239		+ 160		- 149		+ 6.63		- 5.1
48	40	14.7	40	14.2	- 0.240		+ 162		- 149		+ 6.58		+ 0.5
49	39	55.9	39	52.8	- 0.241		+ 163		- 148		+ 6.52		+ 3.1
50	39	37.9	39	43.1	- 0.243		+ 164		- 148		+ 6.50		- 5.2
51	39	0.5	39	3.0	- 0.245		+ 166		- 146		+ 6.38		- 2.5
52	38	36.8	38	36.7	- 0.247		+ 168		- 145		+ 6.32		+ 0.1
53	38	18.6	38	14.6	- 0.249		+ 169		- 145		+ 6.25		+ 4.0
54	38	3.4	37	59.9	- 0.250		+ 170		- 144		+ 6.20		+ 3.5
55	37	46.3	37	42.3	- 0.251		+ 171		- 143		+ 6.15		+ 4.0
57	36	30.6	36	25.8	- 0.255		+ 176		- 141		+ 5.90		+ 4.8
58	35	53.9	35	58.7	- 0.257		+ 177		- 140		+ 5.82		- 4.8
59	35	40.3	35	35.4	- 0.258		+ 179		- 139		+ 5.73		+ 4.9
61	34	49.9	34	49.6	- 0.260		+ 181		- 137		+ 5.58		+ 0.3
62	34	36.8	34	38.5	- 0.261		+ 182		- 136		+ 5.55		- 1.7
63	34	23.9	34	24.4	- 0.262		+ 183		- 136		+ 5.50		- 0.5
64	34	16.5	34	14.1	- 0.263		+ 184		- 135		+ 5.45		+ 2.4
65	33	14.5	33	18.8	- 0.266		+ 186		- 132		+ 5.25		- 4.3
67	31	27.9	31	27.6	- 0.272		+ 192		- 127		+ 4.83		+ 0.3
72	.	.	26	14.5	- 0.284		+ 207		- 111		+ 3.52		.
74	24	46.7	24	41.4	- 0.288		+ 210		- 105		+ 3.08		+ 5.3
75	24	30.1	24	16.3	- 0.290		+ 211		- 103		+ 2.97		+ 13.8
76	22	25.4	22	21.4	- 0.291		+ 216		- 96		+ 2.42		+ 4.0
77	21	43.5	21	50.3	- 0.292		+ 217		- 94		+ 2.28		- 6.8
78	21	44.5	21	41.2	- 0.293		+ 217		- 94		+ 2.23		+ 3.3
79	21	27.7	21	27.8	- 0.293		+ 217		- 93		+ 2.17		- 0.1
80	21	22.7	21	17.8	- 0.293		+ 218		- 92		+ 2.13		+ 4.9
81	21	15.3	21	8.6	- 0.293		+ 218		- 92		+ 2.08		+ 6.7
84	18	21.7	18	16.3	- 0.294		+ 223		- 80		+ 1.23		+ 5.4
93	4	51.9	4	45.9	- 0.290		+ 231		- 21		- 2.82		+ 6.0
94	4	42.0	4	37.2	- 0.290		+ 231		- 20		- 2.85		+ 4.8
95	4	27.2	4	24.6	- 0.289		+ 231		- 20		- 2.90		+ 2.6
96	4	20.3	4	16.8	- 0.288		+ 231		- 19		- 2.95		+ 3.5
99	+ 2	51.2	+ 2	43.0	- 0.285		+ 230		- 12		- 3.40		+ 8.2

PEKING.

No. of Photo.	Observed Position-Angle.		Tabular Position-Angle.						O. — C.
	o	i	o	i	i		i	i	
15	+ 45	9.5	+ 45	15.7	— 0.217 $\delta\lambda_3$	+ 142 δA	— 156 δD	+ 8.58 $\delta\pi$	— 6.2
19		43 43.5	43	46.7	— 0.224	+ 148	— 154	+ 8.48	— 3.2
21		43 9.9	43	6.7	— 0.228	+ 151	— 153	+ 8.45	+ 3.2
22	+ 42	40.7	+ 42	45.0	— 0.230	+ 153	— 153	+ 8.42	— 4.3
44	— 8	2.2	— 8	5.9	— 0.254	+ 215	+ 33	— 4.13	+ 3.7
45	— 8	20.9	— 8	22.4	— 0.253	+ 214	+ 34	— 4.20	+ 1.5
46	— 8	33.2	— 8	39.1	— 0.252	+ 214	+ 35	— 4.28	+ 5.9
49	— 9	40.9	— 9	40.2	— 0.249	+ 212	+ 39	— 4.54	— 0.7
50	— 9	56.3	— 9	58.8	— 0.247	+ 211	+ 40	— 4.60	+ 2.5
51	— 10	14.3	— 10	16.5	— 0.245	+ 210	+ 41	— 4.68	+ 2.2
53	— 10	54.8	— 10	54.7	— 0.243	+ 209	+ 44	— 4.82	— 0.1
54	— 11	14.2	— 11	12.6	— 0.242	+ 208	+ 45	— 4.88	— 1.6
56	— 11	45.5	— 11	47.5	— 0.239	+ 207	+ 47	— 5.02	+ 2.0
57	— 12	4.7	— 12	5.8	— 0.238	+ 206	+ 48	— 5.08	+ 1.1
58	— 12	25.0	— 12	24.1	— 0.237	+ 205	+ 49	— 5.15	— 0.9
59	— 12	38.4	— 12	39.4	— 0.235	+ 204	+ 50	— 5.20	+ 1.0
60	— 12	53.9	— 12	55.6	— 0.233	+ 204	+ 51	— 5.27	+ 1.7
61	— 13	15.2	— 13	12.9	— 0.232	+ 203	+ 52	— 5.32	— 2.3
63	— 13	45.8	— 13	46.3	— 0.230	+ 201	+ 54	— 5.45	+ 0.5
65	— 14	16.0	— 14	16.3	— 0.228	+ 200	+ 55	— 5.55	+ 0.3
67	— 14	55.5	— 14	49.6	— 0.226	+ 198	+ 57	— 5.65	— 5.9
68	— 15	3.0	— 15	3.1	— 0.225	+ 198	+ 58	— 5.70	+ 0.1
69	— 15	18.4	— 15	22.5	— 0.223	+ 197	+ 59	— 5.77	+ 4.1
70	— 15	36.4	— 15	36.4	— 0.222	+ 196	+ 59	— 5.82	0.0
71	— 15	51.9	— 15	49.7	— 0.221	+ 196	+ 60	— 5.85	— 2.2
72	— 15	55.8	— 16	3.2	— 0.220	+ 195	+ 61	— 5.90	+ 7.4

KERGUELEN.

No. of Photo.	Observed Position-Angle.		Tabular Position-Angle.						O. — C.
	o	i	o	i	i		i	i	
7	+ 37	3.7	+ 36	58.4	— 0.244 $\delta\lambda_4$	+ 166 δA	— 136 δD	+ 1.25 $\delta\pi$	+ 5.3
17	+ 2	4.1	+ 2	3.5	— 0.269	+ 220	— 9	+ 3.90	+ 0.6
18	+ 1	13.5	+ 1	3.4	— 0.267	+ 219	— 5	+ 3.90	+ 10.1
19	+ 0	48.6	+ 0	44.3	— 0.266	+ 219	— 3	+ 3.88	+ 4.3
20	+ 0	14.0	+ 0	17.4	— 0.265	+ 219	— 1	+ 3.90	— 3.4
25	— 6	17.3	— 6	26.0	— 0.248	+ 210	+ 26	+ 3.63	+ 8.7
32	— 10	34.3	— 10	42.3	— 0.232	+ 201	+ 41	+ 3.37	+ 8.0
33	— 17	0.7	— 17	9.7	— 0.207	+ 184	+ 62	+ 2.85	+ 9.0

HOBART TOWN.

No. of Photo.	Observed Position-Angle.	Tabular Position-Angle.						O. — C.
		o	i	i			i	
9	+ 2 34.4	+ 2 23.5	— 0.275 $\delta \lambda_6$	+ 222 δA	— 10 δD	— 5.63 $\delta \pi$	[+ 10.9]	
10	+ 2 15.2	+ 2 14.0	— 0.273	+ 222	— 10	— 5.63	+ 1.2	
11	+ 2 3.7	+ 2 3.3	— 0.272	+ 222	— 9	— 5.63	+ 0.4	
12	+ 2 6.5	+ 1 55.6	— 0.272	+ 222	— 8	— 5.63	+ 10.9	
13	+ 1 36.8	+ 1 33.3	— 0.271	+ 222	— 7	— 5.63	+ 3.5	
14	+ 1 5.7	+ 1 5.1	— 0.271	+ 221	— 4	— 5.63	+ 0.6	
15	+ 0 58.5	+ 0 52.3	— 0.268	+ 221	— 4	— 5.63	+ 6.2	
16	+ 0 31.2	+ 0 26.5	— 0.267	+ 221	— 2	— 5.63	+ 4.7	
17	+ 0 28.3	+ 0 21.8	— 0.267	+ 221	— 2	— 5.63	+ 6.5	
18	+ 0 6.7	— 0 0.9	— 0.267	+ 220	0	— 5.63	+ 7.6	
19	+ 0 4.1	— 0 8.9	— 0.266	+ 220	+ 1	— 5.62	+ 13.0	
22	— 1 32.0	— 1 39.6	— 0.264	+ 219	+ 7	— 5.60	+ 7.6	
23	— 1 45.4	— 1 51.2	— 0.262	+ 218	+ 8	— 5.58	+ 5.8	
24	— 1 59.4	— 2 1.5	— 0.262	+ 218	+ 9	— 5.57	+ 2.1	
25	— 2 12.7	— 2 13.4	— 0.262	+ 218	+ 9	— 5.58	+ 0.7	
26	— 2 15.0	— 2 25.8	— 0.260	+ 217	+ 10	— 5.58	+ 10.8	
27	— 2 37.1	— 2 52.3	— 0.260	+ 217	+ 12	— 5.57	+ 15.2	
28	— 3 3.2	— 3 6.0	— 0.259	+ 216	+ 13	— 5.57	+ 2.8	
29	— 3 16.0	— 3 20.9	— 0.258	+ 216	+ 14	— 5.57	+ 4.9	
30	— 3 28.0	— 3 29.6	— 0.258	+ 216	+ 14	— 5.55	+ 1.6	
32	— 3 58.8	— 3 55.8	— 0.257	+ 215	+ 16	— 5.53	— 3.0	
33	— 3 51.4	— 4 0.4	— 0.257	+ 215	+ 16	— 5.55	+ 9.0	
34	— 4 4.6	— 4 11.1	— 0.256	+ 215	+ 17	— 5.53	+ 6.5	
35	— 4 11.9	— 4 20.5	— 0.256	+ 215	+ 18	— 5.52	+ 8.6	
36	— 4 37.2	— 4 33.9	— 0.255	+ 214	+ 19	— 5.52	— 3.3	
37	— 4 27.3	— 4 39.7	— 0.255	+ 214	+ 19	— 5.52	+ 12.4	
38	— 4 52.2	— 4 55.7	— 0.254	+ 214	+ 20	— 5.52	+ 3.5	
39	— 5 2.4	— 5 10.4	— 0.253	+ 213	+ 21	— 5.50	+ 8.0	
40	— 5 21.6	— 5 21.4	— 0.251	+ 213	+ 22	— 5.48	— 0.2	
41	— 5 36.8	— 5 42.6	— 0.251	+ 212	+ 23	— 5.48	+ 5.8	
42	— 5 43.9	— 5 54.0	— 0.249	+ 212	+ 24	— 5.47	+ 10.1	
43	— 6 1.9	— 6 4.0	— 0.248	+ 212	+ 24	— 5.47	+ 2.1	
44	— 6 12.9	— 6 16.2	— 0.247	+ 211	+ 25	— 5.45	+ 3.3	
45	— 6 20.0	— 6 25.9	— 0.246	+ 211	+ 26	— 5.43	+ 5.9	
46	— 6 38.0	— 6 40.3	— 0.246	+ 211	+ 27	— 5.43	+ 2.3	
47	— 6 43.7	— 6 54.5	— 0.246	+ 210	+ 28	— 5.43	+ 10.8	
48	— 6 58.7	— 7 9.2	— 0.246	+ 210	+ 29	— 5.42	+ 10.5	

CAMPBELL TOWN.

No. of Photo.	Observed Position-Angle.		Tabular Position-Angle.					O.—C.	
	o	i	c	i	i		i		
10	+ 10	47.1	+ 10	46.7	- 0.286 $\delta \lambda_0$	+ 224 δA	- 46 δD	- 5.50 $\delta \pi$	+ 0.4
11	+ 9	49.0	+ 9	47.7	- 0.285	+ 224	- 42	- 5.55	+ 1.3
12	+ 9	43.1	+ 9	38.0	- 0.285	+ 224	- 41	- 5.55	+ 5.1*
17	+ 7	10.4	+ 7	4.1	- 0.282	+ 225	- 30	- 5.65	+ 6.3
18	+ 6	47.5	+ 6	47.9	- 0.282	+ 224	- 29	- 5.65	- 0.4
19	+ 6	45.4	+ 6	33.0	- 0.282	+ 224	- 28	- 5.65	+ 12.4
23	+ 4	46.5	+ 4	42.4	- 0.279	+ 224	- 20	- 5.70	+ 4.1
24	+ 4	45.4	+ 4	28.0	- 0.277	+ 223	- 19	- 5.70	+ 17.4
28	+ 0	9.4	+ 0	1.5	- 0.269	+ 220	0	- 5.73	+ 7.9
29	- 0	48.0	- 1	0.5	- 0.265	+ 219	+ 4	- 5.72	+ 12.5
31	- 2	15.2	- 2	21.5	- 0.262	+ 218	+ 10	- 5.70	+ 6.3
32	- 2	26.4	- 2	41.3	- 0.260	+ 217	+ 11	- 5.68	+ 14.9
33	- 3	17.7	- 3	10.7	- 0.258	+ 216	+ 13	- 5.68	- 7.0
34	- 3	37.0	- 3	43.2	- 0.256	+ 216	+ 15	- 5.67	+ 6.2
35	- 4	12.8	- 4	24.9	- 0.255	+ 215	+ 18	- 5.63	+ 12.1
36	- 4	45.2	- 4	56.0	- 0.253	+ 214	+ 20	- 5.62	+ 10.8
37	- 5	12.3	- 5	13.1	- 0.253	+ 213	+ 21	- 5.62	+ 0.8
38	- 5	18.7	- 5	25.7	- 0.252	+ 213	+ 22	- 5.60	+ 7.0
39	- 5	44.5	- 5	52.0	- 0.251	+ 212	+ 24	- 5.58	+ 7.5
40	- 6	10.2	- 6	12.0	- 0.249	+ 212	+ 25	- 5.57	+ 1.8
41	- 6	27.7	- 6	26.8	- 0.248	+ 211	+ 26	- 5.57	- 0.9
42	- 6	31.0	- 6	44.5	- 0.247	+ 211	+ 27	- 5.55	+ 13.5
44	- 7	14.7	- 7	18.6	- 0.246	+ 210	+ 29	- 5.52	+ 3.9
45	- 7	30.7	- 7	40.3	- 0.244	+ 209	+ 31	- 5.50	+ 9.6
46	- 7	52.0	- 7	59.0	- 0.242	+ 208	+ 32	- 5.48	+ 7.0
47	- 8	14.4	- 8	23.3	- 0.240	+ 207	+ 33	- 5.47	+ 8.9
48	- 12	50.2	- 12	50.4	- 0.225	+ 197	+ 49	- 5.17	+ 0.2
49	- 13	28.4	- 13	19.3	- 0.222	+ 196	+ 50	- 5.12	- 9.1
50	- 13	35.3	- 13	33.1	- 0.221	+ 195	+ 51	- 5.10	- 2.2
51	- 13	41.4	- 13	47.8	- 0.220	+ 195	+ 52	- 5.08	+ 6.4
52	- 13	55.8	- 13	58.9	- 0.219	+ 194	+ 52	- 5.07	+ 3.1
53	- 14	15.5	- 14	12.9	- 0.218	+ 193	+ 53	- 5.05	- 2.6

* Time uncertain.

QUEENSTOWN.

No. of Photo.	Observed Position-Angle.	Tabular Position-Angle.						O.—C.
		o	'	''	''	''	''	
114	+ 40 47.2	+ 40	50.7	— 0.232 $\delta \lambda_7$	+ 155 δA	— 145 δD	— 5.23 $\delta \pi$	— 3.5
115	+ 40 26.4	+ 40	24.6	— 0.233	+ 156	— 144	— 5.30	+ 1.8
116	+ 40 2.9	+ 40	0.5	— 0.233	+ 158	— 143	— 5.35	+ 2.4
117	+ 39 45.8	+ 39	45.0	— 0.235	+ 158	— 143	— 5.38	+ 0.8
118	+ 39 16.9	+ 39	16.1	— 0.236	+ 160	— 142	— 5.45	+ 0.8
119	+ 38 58.9	+ 38	55.7	— 0.238	+ 161	— 142	— 5.48	+ 3.2
120	+ 38 8.9	+ 38	17.3	— 0.240	+ 164	— 140	— 5.58	— 8.4
122	+ 37 7.2	+ 37	6.9	— 0.244	+ 168	— 138	— 5.73	+ 0.3
123	+ 36 38.6	+ 36	36.7	— 0.246	+ 169	— 137	— 5.80	+ 1.9
124	+ 36 5.4	+ 36	2.6	— 0.248	+ 171	— 135	— 5.87	+ 2.8
125	+ 35 28.3	+ 35	31.2	— 0.250	+ 173	— 134	— 5.92	— 2.9
126	+ 34 49.5	+ 34	49.1	— 0.252	+ 175	— 132	— 6.02	+ 0.4
127	+ 34 14.6	+ 34	12.7	— 0.254	+ 177	— 131	— 6.08	+ 1.9
128	+ 33 41.4	+ 33	41.6	— 0.255	+ 179	— 129	— 6.15	— 0.2
129	+ 33 12.1	+ 33	8.0	— 0.257	+ 181	— 128	— 6.22	+ 4.1
130	+ 32 20.8	+ 32	17.5	— 0.259	+ 183	— 126	— 6.30	+ 3.3
131	+ 31 59.5	+ 31	54.0	— 0.261	+ 185	— 125	— 6.35	+ 5.5
132	+ 31 25.2	+ 31	25.4	— 0.262	+ 186	— 123	— 6.40	— 0.2
133	+ 30 59.2	+ 31	1.4	— 0.264	+ 187	— 122	— 6.45	— 2.2
134	+ 30 38.1	+ 30	33.6	— 0.265	+ 188	— 121	— 6.50	+ 4.5
135	+ 30 13.1	+ 30	6.3	— 0.266	+ 190	— 119	— 6.55	+ 6.8
139(?)	+ 22 27.4	+ 22	30.3	— 0.280	+ 208	— 94	— 7.20	— 2.9
142	+ 22 55.5	+ 22	51.5	— 0.280	+ 208	— 95	— 7.18	+ 4.0
143	+ 21 54.6	+ 21	55.4	— 0.281	+ 209	— 91	— 7.23	— 0.8
144	+ 22 18.9	+ 22	13.1	— 0.281	+ 209	— 93	— 7.22	+ 5.8
145	+ 21 30.7	+ 21	25.1	— 0.281	+ 210	— 89	— 7.27	+ 5.6
151	+ 6 31.8	+ 6	27.7	— 0.280	+ 224	— 28	— 7.50	+ 4.1
153	+ 6 14.4	+ 6	22.3	— 0.278	+ 224	— 27	— 7.50	— 7.9
154	+ 6 19.5	+ 6	7.8	— 0.277	+ 223	— 26	— 7.48	+ 11.7
155	+ 5 47.4	+ 5	41.3	— 0.276	+ 223	— 24	— 7.47	+ 6.1
156	+ 4 46.2	+ 4	44.6	— 0.275	+ 223	— 20	— 7.43	+ 1.6
158	+ 4 1.9	+ 4	2.2	— 0.274	+ 223	— 17	— 7.40	— 0.3
159	+ 3 55.1	+ 3	37.9	— 0.273	+ 223	— 15	— 7.37	+ 17.2
160	+ 3 14.1	+ 3	5.1	— 0.273	+ 222	— 13	— 7.35	+ 9.0
161	+ 3 2.2	+ 2	54.0	— 0.272	+ 222	— 12	— 7.33	+ 8.2
163	+ 0 10.4	+ 0	16.3	— 0.266	+ 220	— 1	— 7.15	— 5.9
164	— 0 22.5	— 0	9.9	— 0.264	+ 220	+ 1	— 7.12	— 12.6
165	— 0 19.8(?)	— 0	22.9	— 0.264	+ 219	+ 2	— 7.10	+ 3.1(?)
166	— 0 35.0	— 0	44.4	— 0.263	+ 219	+ 3	— 7.07	+ 9.4
167	— 1 2.8	— 1	4.0	— 0.262	+ 219	+ 4	— 7.03	+ 1.2
171	— 2 48.0	— 2	56.1	— 0.259	+ 216	+ 12	— 6.87	+ 8.1
172	— 3 41.9	— 3	44.2	— 0.256	+ 215	+ 15	— 6.78	+ 2.3
173	— 4 13.2	— 4	10.7	— 0.254	+ 214	+ 17	— 6.73	— 2.5
176	— 8 39.5	— 8	39.8	— 0.235	+ 206	+ 34	— 6.18	+ 0.3
177	— 8 40.1	— 8	46.9	— 0.235	+ 206	+ 35	— 6.17	+ 6.8

CHATHAM ISLAND.

No. of Photo.	Observed Position-Angle.	Tabular Position-Angle.						O.—C.
		°	'	''				
15	+ 29 26.6	+ 29	29.7	— 0.267 $\delta\lambda_8$	+ 191 δA	— 118 δD	— 8.08 $\delta\pi$	— 3.1
16	+ 29 19.7	+ 29	16.0	— 0.267	+ 192	— 117	— 8.10	+ 3.7
17	+ 28 55.4	+ 28	59.1	— 0.267	+ 193	— 116	— 8.13	— 3.7
24	+ 16 8.7	+ 16	6.0	— 0.284	+ 218	— 68	— 8.85	+ 2.7
25	+ 15 42.9	+ 15	46.4	— 0.283	+ 219	— 67	— 8.85	— 3.5
27	+ 13 28.0	+ 13	16.9	— 0.282	+ 221	— 57	— 8.83	+ 11.1
29	+ 9 22.3	+ 9	15.9	— 0.281	+ 223	— 39	— 8.72	+ 6.4

CHAPTER IV.

OPTICAL OBSERVATIONS OF THE TRANSIT.

The optical observations relating to the Transit of Venus, included in the programme of the Commission, were as follows :

1. Observations of the moment at which the indentation made by Venus in entering upon the limb of the Sun first became visible.
2. Observations of the distance apart of the cusps, during the ten minutes preceding first internal contact, made with the AIRY-VALZ double-image micrometer.
3. Observations of the time of internal contact.
4. Measures of the thickness of the strip of light between the limbs of Venus and of the Sun during a few minutes following internal contact.
5. At the egress of the planet, a series of observations and measures in reverse order made in the same manner.
6. Observations of an artificial Transit of Venus, made for the double purpose of practicing the observers and of determining their personal equations in observations of this class.

§ 1. REMARKS ON OBSERVATIONS OF THE ARTIFICIAL TRANSIT OF VENUS.

In describing and discussing the observations, we shall begin with those made on the artificial transit, because they are expected to be used in discussing the subsequent observations. They may be divided into two classes, of which the objects are slightly dissimilar, those made before the parties went out and those made after their return.

The principal object of the first class was the practice of the observers and the ascertainment of the errors to which the observations were liable. As a general rule, the observed times were compared with the actual times noted by an observer immediately behind the model transit, but the observers frequently practiced themselves without having any such record.

After the return of the parties, the object aimed at was to secure an observation as much as possible like that actually made at the station, and to ascertain its error. This was a very desirable object to attain, and one which must be considered worth attempting in all such observations. Many practical difficulties were, however, encountered in carrying it out, one being that several of the observers did not return to Washington for a long period after their observations. The results are, however, presented in full in the following section.

Among the general results concluded from the observations of the model transit were the following:

That external contacts admit of fully as accurate observation and comparison as internal ones, if allowance be made for the fact that the contact must become sensible before being seen; that the accuracy of the observations is gradually increased by practice on the part of the observer; that there is only one definite phase at internal contact which can be clearly differentiated, namely, one which, with slight accidental and personal differences, corresponds to the moment of true contact. The formation of the black drop and the succession of phenomena described by various observers all run into each other by insensible gradations, and therefore do not admit of distinctive observations. The following more special statement of conclusions to be drawn from the observations is principally extracted from a paper in the Monthly Notices of the Royal Astronomical Society for March, 1877 (volume 37, page 237):

(1) A defect of much of the reasoning on this subject is this: It has too generally been assumed that the geometric outlines of Venus and the Sun, considered as mathematical lines, can be noted in observation with the same sort of definiteness and precision as that with which the mind conceives them, and sufficient attention has not been paid to the practical difficulties which the eye meets with in representing this geometric conception. I conceive that the question whether a certain phase can or cannot be definitely distinguished and observed by the eye is to be settled by actual trial and by a consideration of the imperfections of vision rather than by a consideration of its purely geometric definiteness of form.

(2) One result of the trials with the artificial transit is that there is a certain phase near that of external contact which can be observed with the same order of precision as the internal contact, provided that the proper conditions are fulfilled. Among these conditions are that the observer shall previously have practiced on the artificial transit; that he shall know at exactly what point of the Sun's limb to look for the first contact; that he shall know when to look for the contact with an uncertainty of not much less than half a minute nor much more than a minute; and that he shall have a telescope of fixed size and power. Of course the phase thus observed will not be geometric contact, but that occurring at the time when the notch in the Sun's limb first became visible. This phase varies much less with variations in the atmospheric condition and in the size and power of the telescopes than might have been supposed.

(3) The phase of external contact at egress is more uncertain than at ingress, owing to the doubt of the observer as to whether the notch has or has not disappeared from his view, that doubt extending over a longer period than the doubt as to when he first sees the notch at ingress.

(4) In describing the phenomena near the time of internal contact, I shall consider the planet to be approaching egress. For reasons which I shall soon mention, the artificial transit was placed at a distance of about eleven hundred yards from the place of observation, so that a greater or less amount of atmospheric undulation was always present. Supposing, then, that the planet was approaching the Sun's limb, and, the thread of light growing thin owing to the approach of contact, the first thing which an observer would remark was that, in consequence of the constant changes of outline, caused by atmospheric disturbance of the images, no set of phenomena could be described as invariable. It would be necessary to combine judgment with sight by considering what might be called a mean phenomenon. Different observers might form different judgments as to what this mean phenomenon was. It would, however, always be seen that, as the thread of light became quite thin, it looked darker than the rest of the Sun, and unless the atmosphere was more steady than is usual in the day-time the line of light would occasionally be broken up into two or more very irregular threads or twists of light, which would gradually grow fainter until they would occasionally almost disappear from view. Repeated trials showed that the time of true internal contact was marked by the moment at which light entirely ceased to glimmer across the dark space formed by the approach of the limb of the planet to that of the Sun.

(5) Every attempt to estimate an apparent contact, or moment at which the limbs of Venus and the Sun were tangent to each other, without reference to the appearance of the thread of light, was a total failure. It was, in fact, impossible to make any such estimate without an uncertainty of half a minute or more. This will not appear surprising if we reflect that the outlines of Venus and the Sun cannot present themselves to the vision as geometric lines, but only as more or less indefinite edges of a visible surface of sunlight, which visible surface disappears most gradually near the region of contact, and, in fact, at the moment of ideal "apparent contact" cannot be seen at all at the point of contact. Any one who wishes to satisfy himself on this point has only to examine a series of figures in which the black drop is represented, and try to decide which of them represents apparent contact. If he wishes to come as near nature as possible, he must shade off the outlines both of Venus and the Sun so that they shall terminate in a soft border, and view the picture through a rising current of hot air.

(6) Any artificial representation of these phenomena in which the bright surface of the Sun is surrounded by a dark background, must fail to be correct in a very important particular. As a matter of fact, we know that the

atmosphere immediately around the Sun's limb is of dazzling brilliancy. In meridian observations of the Sun, when the light is so cut down by a dark glass as no longer to dazzle the eye, the fine spider lines are visible on the background of the atmosphere, else it would be impossible to observe the transit of the Sun's first limb. This brilliancy of the atmosphere must greatly diminish irradiation, and if the Sun is observed through haze (as must often be the case in observations of contact when the Sun is near the horizon) irradiation may be entirely destroyed. It will therefore be impossible to observe an actual internal contact of Venus or Mercury with a precision corresponding to that of an artificial contact on a black background.

(7) The atmosphere affects the phenomena of contact in three ways :

a. By illuminating the background, as just described;

β. By producing undulations in the outlines of the images, thereby preventing the phenomena from being invariable;

γ. By softening the outline of the Sun's limb and thus rendering it more or less indefinite.

The artificial transit to which I have alluded was placed at a distance, in order that all these effects might be produced and studied, otherwise the optical phenomena of contact may be entirely different. For instance, because the black drop is sharply seen in the artificial transit when the background is quite black, it does not follow that it will be noticed in the actual transit.

(8) I have said little of the black drop, partly because I do not think there can be much room for a real difference of opinion respecting its nature and causes, and partly because the question whether it is seen is of entirely secondary importance except as affording some indication of the sharpness of definition. In looking at the artificial transit it was very easy, about the moment of internal contact, by taking a general mean outline of the undulating imago of the planet, and imagining that outline continued across the undulating line of light and darkness mixed, into which the ideal thread was reduced by the imperfections of vision, to see something like a black drop. On the other hand another observer, with his attention fixed solely on the thread of light, would see nothing of the sort. The final disappearance of the thread of sunlight being the only phase which can be actually observed, an observer, fixing his attention exclusively on this, would not see any black drop at all, unless the amount of irradiation was exceptionally great.

(9) The general conclusion to which I am led is that there is but one phase of each contact which can be observed with any approach to accuracy, namely :

a. The time when the notch made by Venus advancing on the Sun becomes visible;

β. The time at which true sunlight is first seen all the way around the following limb of the planet;

γ. The planet approaching egress, the time when it first completely cuts off the true limb of the Sun, and the space connecting the limb of the planet with the sky becomes as dark as the planet itself;

δ. The time when the last limb of the planet, leaving the limb of the Sun, disappears from view.

If an observer, at the time of internal contact, does not note, or try to note, the phases *β* and *γ*, there is no definite phase to which his observation can be referred. The old belief that at second internal contact there is a sudden formation of the black drop, which marks the moment of true contact, seems to be entirely disproved. The more clear and dark the atmosphere around the Sun the more rapidly will contact appear to form, whether a black drop is seen or not, but under no circumstances under which an actual transit is likely to be observed for parallax will it be really sudden. When it appears so it is only because the observer fails to notice the gradual darkening and breaking up of the thread of light. From the commencement of this darkening and diffusion, until the "apparent contact," which comes last of all, there are a series of progressive changes, which may extend over a period ranging anywhere from twenty or thirty to ninety seconds, at any point of which a random observation of internal contact may fall. The worse the definition and the lower the planet the greater the range, but the time of true contact is always near the mean of the period.

§ 2. PRACTICE ON THE MODEL TRANSIT.

From what has just been said, it will be seen that an arrangement of the model transit in which vision on the bright cusps should be perfectly sharp, with a perfectly dark background to represent the sky around the Sun and over Venus, would fail to present to the eye phenomena like those of the actual transit in at least three points :

1. When observed with a sufficiently high power, 200 or upward, the limb of the Sun is always subject to more or less atmospheric undulation, especially when the Sun is not near the zenith.

2. The outline of the limb is never perfectly hard, but must always be more or less softened, owing to action of our atmosphere and, perhaps, to that of the solar atmosphere also.

3. The background around the Sun is not black, but is so highly illuminated by the Sun's rays as to be clearly visible even through a dark glass which reduces the Sun's light to such an extent as to make it entirely pleasant to the eye.

To attain the conditions of undulating images, a softened outline and a slightly illuminated background, the artificial transit was placed on Winder's Building, an elevated structure about 3,300 feet from the Observatory. The line of sight from the dome of the Observatory is from 60 to 80 feet above the ground, over a part of the city where there are but few buildings. The amount of disturbance of images from atmospheric undulations is very varied, being greatest when the Sun is shining during a summer's day. An average amount of undulation, equal to that of the Sun as ordinarily viewed at moderate altitudes, is easily attained by selecting an appropriate time of day and condition of the atmosphere for the experiments.

The essential features of the apparatus are a triangular opening through a blackened disk of metal, through which a white screen is viewed. The contrast between the screen and the metal appears about the same as between the Sun and the surrounding sky under ordinary atmospheric conditions. The inclined sides of the triangle are at an inclination about equal to that of the path of Venus to the limb of the Sun as seen from a southern station during the transit of 1874. The diameter of the disk which represents Venus is twelve inches, and the angle subtended is nearly that subtended by the planet during the transit. It is made of sheet-metal, painted black, and moves immediately behind the triangle, the sides of which represent the limb of the Sun. To keep it in place, a second triangle is built immediately behind the first, the space between the two being just sufficient to allow the disc to slide along. This second triangle is slightly larger than the first, so as to be invisible from the dome of the Observatory. The Venus is therefore entirely invisible until it emerges from behind the internal edge of the triangle, when it is seen by projection against the white screen beyond.

The time occupied in passing from external to internal contact is 32 minutes. If the disk be allowed to run on, it will reach second internal contact in less than 5 minutes after the first, and 32 minutes more will bring it to second external contact.

The actual times of the geometric contacts were observed by an assistant alongside the instrument. At first he noted the times by holding his eye in the plane of the Observatory and the edge of the metal from which the disc emerged, but in the later observations he was stationed behind the screen, and observed the contact through a minute hole on a line from the Observatory to the point of contact.

Observations on this apparatus from the dome of the Observatory were commenced in May, 1873, before the telescopes with which the actual transit was observed were obtained. The instruments used in the early observations were:

(1.) Observatory equatoreal of 9.6 inches aperture, sometimes reduced to 5 inches, with magnifying power of 130. This is designated as E.

(2.) Telescope of 5 inches aperture, belonging to Miss Mitchell; powers 80 to 200. This is designated as M.

(3.) Comet-seeker of 4 inches aperture; power 40. This is designated as C.

The results of these observations are given in the form of errors of observation, or excesses of the times noted by the observers over the true moments of geometric contact. The chronometer comparisons, and other original data on which these results depend, are considered too simple to render their reproduction necessary.

The contacts are designated in their order as I, II, III, IV, the first and last being the external ones; the other two the internal ones.

The first observations were made June 12, 1873, on internal contacts only, there being three observers, Professors NEWCOMB, HALL, and HARKNESS, who alternated at each of the three instruments. The resulting errors of observation, in seconds of time, are as follows; the letters following the times indicate the instruments used:

Second contact.			Third contact.		
Newcomb.	Harkness.	Hall.	Newcomb.	Harkness.	Hall.
s. + 5 M	s. + 5 C	s. . . .	s. - 9 M	s. - 8 C	s. . . .
- 17 M	- 19 C	. . .	- 3 M	- 7 C	. . .
- 2 C	- 10 M	- 12 E	- 9 C	+ 5 M	+ 5 E
- 7 C	- 18 M	- 16 E	- 18 C	0 M	+ 4 E
- 13 E	- 16 C	. . .	- 3 E	- 3 C	. . .
- 3 E	- 12 C	. . .	- 5 E	- 4 C	. . .
- 15 M	+ 2 E	+ 32 C	- 3 M	+ 1 E	. . .
- 19 M	- 4 E	- 27 C	0 M	- 3 E	. . .
- 5 C	- 2 E	- 14 M	- 13 C	- 3 E	. . .
+ 5 C	0 E	- 2 M	- 9 C	- 4 E	. . .

June 13.

First contact.		Second contact.		Third contact.	
Newcomb.	Hall.	Newcomb.	Hall.	Newcomb.	Hall.
s. + 45 M	s. . . .	s. - 9 M	s. - 29 E	s. - 2 M	s. + 7 E
+ 44: M	+ 71: E	- 7	- 28	- 5	+ 1
+ 34	+ 25 E*	- 7	- 26	- 2	+ 2
+ 22	+ 20	- 6	- 8	- 3	+ 3
+ 11	+ 26
+ 14	+ 22

The entire minutes of HALL's observations are sometimes doubtful, as he did not record the times himself.

June 14.

First contact.			Second contact.			Third contact.			Fourth contact.	
Newcomb.	Hall.	Harkness.	Newcomb.	Hall.	Harkness.	Newcomb.	Hall.	Harkness.	Newcomb.	Hall.
s. + 14 M	s. . . .	s. . . .	s. + 4 M	s. 0 E	s. - 10 C	s. + 12	s. + 4	s. + 4	s. . . .	s. - 30 E
+ 15 M	. . .	+ 17 C	- 8 M	- 1 E	+ 1 C	+ 2	+ 8	+ 8	+ 2 M	- 21 E
.	+ 26 C	0 M	- 1 E	+ 2 C	+ 3	+ 4	+ 3	- 5 M	- 24 E
. . .	+ 22 E	. . .	5 M	- 3 E	- 2 C	+ 6	+ 2	+ 7	- 4	- 22 E
. . .	+ 23 E	+ 36 C	+ 6	+ 8	+ 3
. . .	+ 26 E

* Time increased by 1 minute.

The foregoing observations are given to illustrate the large errors to which contact observations are liable when undertaken without previous practice. It must, however, be added that the images were generally very bad, owing to the atmospheric undulations of a summer's day.

In the spring of 1874, previous to the departure of the expeditions, as many of the observers as practicable were engaged in practice with the instrument from day to day; the results showing a gradual improvement in the accuracy with which the contacts could be observed. The exhibition of the results of these observations is not deemed necessary here, as it is believed that any corrections for personal error should rather depend on trials made after the return of the parties.

The Pekin party erected a somewhat similar instrument for their own practice while at the station, and the Japan party painted artificial representations of the planet at and near the times of interval contact, which they studied through their telescopes.

After the return of the parties home, the probable corrections to be applied to the observations of contacts were investigated. Most of the observers who had observed a contact afterward came to Washington and made observations of the artificial transit with the same telescope and eye-piece which they had employed in the actual observation. Efforts were made to have the state of the images, the degree of illumination, and the aspects of the phenomenon in general as nearly as possible like they were in the actual transit. As a general rule, it was found by the observers that on an average sunny day the images of the artificial object were more disturbed than they had been in the actual transit; so that it was sometimes necessary to observe the artificial transit during a partially cloudy day, or to wait till near the time of sunset in order to have sufficiently good images.

We present the observations in regular chronological order. There is, however, one important remark to be made respecting the first four series of the following observations, including those made during the last week of August, 1875. It was found that the observers reported the image of the artificial Venus as seen near midday to be decidedly worse than was that of the actual Venus. To have sufficiently good images it was necessary to wait until nearly 5 o'clock in the afternoon, and the phenomenon was then interfered with by the shadow of the apparatus being thrown upon the white screen. To remedy this at a moment's notice, a sheet of white paper was inserted immediately behind the artificial Venus, so that the latter touched it as it passed along. The result of this was that if the Sun was not shining, which was sometimes the case, those portions of the white paper bordering immediately on the limbs of the Sun and Venus were appreciably darker than the rest of the solar disc, and the apparent occurrence of second contact might therefore be delayed and that of third contact accelerated. If the Sun was shining, it was found that in case the position of the actual Sun was such that the shadow of the straight edge representing the border of the solar disc was visible to the observer, it was the contact with the shadow that was seen, and the observer at the instrument therefore recorded these contacts for comparison.

The general probable result of this defective arrangement on the contacts may be mentioned. The observation of first contact could hardly be affected, as the

position occupied by the real Sun during the observations was nearly in the plane of the straight edge and the observer. Still, when the Sun was not shining, there was a possibility that the slight darkening which surrounded the artificial Venus might result in accelerating this contact. The second and third contacts would probably be correctly observed in sunlight, but might have been noted too soon when the Sun was not shining. The fourth contacts would be entirely uncertain in sunlight, but would probably be nearly correct in the shade.

These suspicious observations are presented in full, with a view of facilitating any judgment upon the reliability of their results.

SERIES I. 1875, AUGUST 25.

Observer, Commander G. P. RYAN, U. S. Navy, who observed first external contact of Venus at Kerguelen very sharply. Errors of his observations of the corresponding contact with the artificial transit were:

No. in order.	Δt s.	No. in order.	Δt s.
1	+ 12.8	8	+ 6.0
2	+ 25.0	9	+ 6.0
3	+ 7.5	10	+ 7.0
4	+ 6.6	11	+ 3.9
5	+ 1.9	12	+ 5.5
6	+ 8.5	13	+ 3.9
7	+ 5.5	14	+ 5.5

The first observations were not good from inferior images and want of preparation. "The last two are very good, as nearly as possible like the Kerguelen observations, though I think two or three seconds later, perhaps."

Taking this opinion of the observer, and rejecting his opinion of the observation being late, as clearly impossible, the correction to reduce his observation to geometrical contact would be $-4^s.7$. Taking the mean of all except the first two, the correction would be $-5^s.7$. But, for a cause we shall presently mention, these observations are not entirely reliable.

SERIES 2. AUGUST 25.

Capt. C. W. RAYMOND, Corps of Engineers, U. S. Army, who observed the third contact at Campbelltown, Tasmania, observed the corresponding contact with the artificial transit, as follows:

No. in order.	Δt s.	No. in order.	Δt s.
1	- 3.5	6	- 3.0
2	- 2.5	7	- 1.0
3	+ 0.5	8	- 0.5
4	- 1.5	9	+ 0.5
5	- 1.0	10	0.0
		11	- 4.0
Mean	- - - -	- - - -	- 1.5

The mean of these results indicate that Captain RAYMOND'S observed time of third contact requires the correction

$$+ 1^{\text{s}}.5.$$

SERIES 3. AUGUST 30.

Observers, Prof. A. HALL, U. S. Navy, who observed the transit at Wladiwostok, Siberia, and Prof. C. A. YOUNG, of Dartmouth College, who observed at Pekin.

No. in order.	First Contacts.		No. in order.	Second Contacts.	
	Δt HALL. s.	Δt YOUNG. s.		Δt HALL. s.	Δt YOUNG. s.
1	+ 2.7	+ 4.2	1	- 0.2	+ 10.3
2	+ 1.2	- 3.3	2	+ 7.3	+ 8.8
3	+ 5.2	+ 4.2	3	+ 10.4	+ 10.1
4	+ 3.3		4	+ 6.9	+ 7.4
5	+ 3.8	+ 5.8	5	+ 9.5	+ 8.5
6	+ 6.3	- 1.7	6	+ 6.6	+ 5.1
7	+ 3.4	- 2.6	7	+ 5.6	+ 9.6
8	+ 2.9	+ 4.4	8	+ 6.2	+ 6.7
9	+ 2.9	+ 1.4	9	+ 0.2	- 2.3
	<hr/>	<hr/>	10	+ 2.7	+ 1.2
Mean	+ 3.5	+ 1.6	Mean	+ 5.5	+ 6.5

Third Contact.		Fourth Contact.	
No. in order.	Δt YOUNG. s.	No. in order.	Δt YOUNG. s.
1	- 4.2	1	- 4.3
2	- 7.2	2	- 4.3
3	- 7.6	3	- 4.2
4	- 6.1	4	- 11.1
5	- 10.5	5	- 2.1
6	- 6.5	6	- 1.1
7	- 16.5	7	- 4.0
8	- 16.0	8	- 7.5
9	- 19.9	9	- 5.0
10	- 11.9	10	- 9.5
	<hr/>		<hr/>
Mean	- 10.6	Mean	- 5.3

Observers' notes on these contacts.

CONTACT I.—A fair representation of actual case at Wladiwostok.—HALL.

CONTACT II.—At second contact images were better in artificial transit than in actual one.—HALL. Image quite steady. None of the hesitation which was noted in the actual transit.—YOUNG.

CONTACT III.—The phenomena do not at all resemble those seen by me with this telescope at the transit. There seems to be very little hesitation, and the observed moment seems doubtful not more than one second.—YOUNG.

It will be seen that in three cases out of ten Professor YOUNG noted the first external contact before it really occurred. The explanation of this seeming paradox will be given hereafter in connection with Professor WATSON'S observations.

SERIES 4. AUGUST 31.

Observer, H. C. RUSSELL, Esq., who observed the transit in Australia under the auspices of the Colonial Government, and Professor NEWCOMB.

First Contacts.		Second Contacts.			Third Contacts.		Fourth Contacts.	
No. in order.	Δt RUSSELL. s.	No. in order.	Δt RUSSELL. s.	Δt NEWCOMB. s.	No. in order.	Δt RUSSELL. s.	No. in order.	Δt RUSSELL. s.
1	+ 10.5	1	+ 3.7	+ 6.2	1	- 16.6	1	- 2.9
2	+ 11.5	2	+ 4.2	+ 3.2	2	+ 7.4	2	- 0.4
3	+ 13.0	3	+ 1.8	+ 4.3	3	+ 17.5	3	+ 1.2
4	+ 2.5	4	+ 3.8	+ 1.3	4	+ 0.5	4	- 2.3
5	+ 20.6	5	- 1.7	+ 3.8	5	- 8.5	5	- 2.8
6	+ 10.1	6	- 0.2	+ 4.8	6	lost	6	- 2.6
7	+ 6.1	7	+ 2.8	+ 6.8	7	- 6.4	7	- 4.2
8	+ 9.6	8	+ 2.4	+ 4.4	8	- 3.4	8	- 3.2
9	+ 3.2	9	+ 0.4	+ 5.9	9	+ 3.6	9	- 5.5
10	+ 1.2	10	+ 0.4	+ 2.9	10	+ 3.6	10	- 20.2
							11	- 5.2
Mean	+ 8.8	Mean	+ 1.8	+ 4.4	Mean	- 0.3	Mean	- 4.4

Observer's notes.

CONTACT I.—Sun behind a cloud.—H. C. R.

CONTACT II.—Sun clouded all the time.—H. C. R.

CONTACT III.—(1) A mean of 39^s.5 and 50^s [or of - 22^s.1 and - 11^s.6]; (3) and (4) not good, owing to conversation; (9) contact space smaller than usual.—H. C. R.

CONTACT IV.—Light not strong enough, otherwise observations appeared to be good.

The possible uncertainty arising from the screen being held immediately behind the disc was remarked for the first time during Mr. RUSSELL'S observations. A white sheet placed at a distance of several yards behind the transit was therefore used as a screen in all the following observations, an arrangement which precludes all possibility of errors arising from the shadows of the disc.

SERIES 5. 1875, SEPTEMBER 22.

Observers, Prof. ASAPH HALL, and Mr. O. B. WHEELER, his assistant, who observed the Transit of Venus at Wladiwostok, Siberia.

No. in order.	First Contacts.		No. in order.	Second Contacts.		No. in order.	Third Contacts.	
	Δt HALL. s.	Δt WHEELER. s.		Δt HALL. s.	Δt WHEELER. s.		Δt HALL. s.	Δt WHEELER. s.
1	+ 11.0	+ 28.0	1	—	—	1	— 2.2	— 5.7
2	+ 10.5	+ 20.0	2	—	+ 1.5	2	— 0.8	+ 3.2
3	+ 11.5	+ 13.0	3	—	+ 3.0	3	— 3.8	— 4.3
4	+ 17.0	+ 25.0	4	—	+ 10.5	4	— 0.5	— 4.5
5	+ 12.0	+ 15.0	5	—	+ 4.0	5	— 1.8	— 0.8
6	+ 12.0	+ 12.5	6	—	6	— 4.0	— 3.5
7	+ 15.7	7	+ 2.5	+ 6.0	7	— 2.5	— 5.5
8	+ 7.2	+ 26.7	8	+ 4.0	+ 4.0	8	— 0.5	— 4.5
9	+ 15.8	+ 27.8	9	+ 2.7	+ 7.7(?)	9	— 2.5	— 4.5
10	+ 15.0	+ 22.0	10	+ 3.5	+ 5.3	10	— 2.0	— 3.0
Mean	+ 12.8	+ 21.1	Mean	+ 3.2	+ 4.9	Mean	— 2.1	— 3.3

Observers' remarks.

CONTACT I.—Very bad images.—HALL. By no means as fair an object for observation as that seen at Wladiwostok.—WHEELER.

CONTACT II.—Images fair.—HALL. Much better conditions of atmosphere than during first contacts.—WHEELER.

CONTACT III.—Fair images.

SERIES 6. 1875, SEPTEMBER 22.

No. in order.	Second Contacts repeated.	
	Δt HALL. s.	Δt WHEELER. s.
1	+ 4.5	+ 5.5
2	+ 2.5	+ 12.5
3	+ 2.2	+ 3.2
4	+ 1.7	+ 5.2
5	+ 0.7	— 5.8
6	+ 2.0	+ 5.5
7	+ 1.0	+ 5.5
8	+ 2.0	+ 6.5
9	+ 2.7	+ 2.7
10	+ 2.5	+ 1.5
Mean	+ 2.2	+ 4.2

Observers' remark.

Fair images; (5) best.—WHEELER.

SERIES 7. 1875, SEPTEMBER 23 AND 24.

Same observers; first contacts.

No. in order.	September 23.		No. in order.	September 24.	
	Δt HALL. s.	Δt WHEELER. s.		Δt HALL. s.	Δt WHEELER. s.
1	+ 12.0	+ 21.0	1	+ 6.2	+ 13.8
2	+ 13.0	+ 25.5	2	+ 7.5	+ 12.2
3	+ 11.0	+ 18.5	3	+ 14.5	+ 7.5
4	+ 6.5	+ 17.0	4	+ 5.0	+ 14.5
5	+ 8.0	+ 16.0	5	+ 5.5	+ 6.5
6	+ 7.0	+ 10.5	6	+ 5.7	+ 5.5
7	+ 9.0	+ 13.5	7	+ 8.0	+ 0.7
8	+ 7.5	+ 12.0	8	+ 8.5	+ 10.0
9	+ 8.2	+ 16.7	9	+ 5.5	+ 5.0
10	+ 12.0	+ 26.5	10	+ 5.0
Mean	+ 9.4	+ 17.6	Mean	+ 7.6	+ 8.1

Observers' remarks.

SEPTEMBER 23.—Images fair, but some boiling, and not so sharp as the real contact at Wladiwostok.—HALL. Fair images, but not as well defined as the actual observation at Wladiwostok.—WHEELER.

SEPTEMBER 24.—No. 7 doubtful, eye not being at the telescope soon enough. Images very good, but not quite so sharp as in real contact as observed at Wladiwostok.—HALL. Conditions of atmosphere very favorable. Image very distinct, but not as well defined as the actual observation at Wladiwostok.—WHEELER.

SERIES 8. 1875, OCTOBER 29.

Observers, Commander GEORGE P. RYAN, who repeats observations of August 25, and Professor NEWCOMB.

No. in order.	First Contacts.	
	Δt RYAN. s.	Δt NEWCOMB. s.
1	0.0	+ 3.0
2	+ 11.0	+ 9.0
3	+ 10.0	+ 8.5
4	+ 8.0	+ 7.0
5	+ 7.5	+ 7.5
6	+ 7.3	+ 6.3
7	+ 5.0	+ 4.0
8	+ 4.0
9	+ 9.0
10	+ 11.5
Mean	+ 7.3	+ 6.5

SERIES 9. 1876, JANUARY 14.

Observer, Prof. J. C. WATSON, of Ann Arbor, Michigan, who observed the Transit of Venus at Pekin.

First Contact.		Second Contact.	
No. in order.	Δt WATSON. s.	No. in order.	Δt WATSON. s.
1	+ 9.2	1	- 0.5
2	- 0.8	2	- 0.8
3	- 0.2	3	0.0
4	+ 2.0		
5	+ 6.5		
6	+ 9.0		
7	+ 3.0		
8	- 2.5		
9	- 4.0		
10	- 0.2		
11	+ 1.5		
12	- 1.0		
Mean	+ 2.2		

Observer's remarks.

Telescope disturbed greatly by wind; seeing otherwise good. First contact not quite like real transit as observed at Pekin. Second contact not like what I saw in case of transit. Only one phase of the real contact exhibited.

Before commencing another series, the attention of Professor WATSON was called to the fact that one-half his observations of first contact were recorded before they had occurred, and therefore at a time when the disk representing Venus was absolutely invisible from his station. He explained this by the fact that the image was frequently disturbed in a way which led him to suspect the occurrence of contact, and if, on watching this disturbance, it grew into a well-ascertained contact, the time when it was first noticed was recorded as that of contact. The disturbing influence of the wind added to the difficulty of deciding when the contact should be considered as having been first actually seen.

The fact that these doubtful contacts were incompatible with the description of the actual contact made at Pekin escaped the attention both of Professor WATSON and myself at the time. This will be seen by examining his records of contact observations to be given hereafter.

SERIES 10. 1876, JANUARY 15.

Observer, Professor WATSON.

Second Contacts.		Third Contacts.	
No. in order.	Δt WATSON. s.	No. in order.	Δt WATSON. s.
1	- 5.5	1	+ 9.7
2	- 8.0	2	+ 11.1
3	- 21.6	3	+ 8.8
4	- 10.4	4	+ 8.1
5	- 11.4	5	+ 8.0
6	- 9.7	6	+ 10.1
7	- 13.5	7	+ 9.0
8	- 15.3	8	(+ 5.4)
9	- 14.4	9	+ 6.8
10	10	+ 9.0
11	- 13.0		
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Mean	- 12.3	Mean	+ 9.0

Observer's remarks.

Images very unsteady. Seeing very bad. Although the images were dancing, the appearance was not like the real transit. Only a single phase of it shown. In the case of II or III contact, I observed the formation or ending of a penumbra which surrounded dark image caused by rapid vibration of images.

SERIES 11. 1876, JANUARY 20.

Observers, Professor WATSON, Professor NEWCOMB, who used 5'' telescope (power about 120); and, for second contacts, Professor C. H. F. PETERS, who observed this contact during the Transit of Venus, at Queenstown, New Zealand.

No. in order.	First Contacts.		Second Contacts.		Third Contacts.		Fourth Contacts.
	Δt WATSON. s.	Δt NEWCOMB. s.	Δt WATSON. s.	Δt PETERS. s.	Δt NEWCOMB. s.	Δt WATSON. s.	Δt WATSON. s.
1	+ 2.6	+ 11.6	- 3.2	- 1.2	- 4.7	+ 1.0	- 8.4
2	+ 2.0	+ 14.5	- 8.8	+ 0.7	- 3.8	- 1.5	- 6.5
3	- 1.7	+ 9.3	- 3.6	+ 3.4	- 0.6	+ 0.4	- 7.0
4	+ 0.5	+ 11.0	- 2.6	+ 2.9	- 1.6	- 3.0	- 8.8
5	+ 2.4	+ 13.4	- 0.4	+ 1.1	- 3.9	+ 2.8	- 10.3
6	- 1.2	+ 11.3	- 4.0	0.0	- 0.5	- 2.1	- 9.3
7	+ 4.1	+ 15.1	- 2.4	+ 0.6	- 3.4	- 4.0	- 8.0
8	+ 10.5	+ 15.5	- 1.9	+ 1.6	- 1.9	+ 0.1	- 10.8
9	+ 4.5	+ 9.0	- 2.9	+ 1.1	- 0.4	+ 4.7	- 8.4
10	- 5.3	+ 2.6
11	+ 3.5	+ 15.5	+ 1.4
12	+ 2.1	+ 9.1	- 2.2
13	- 1.5
14	- 5.4
	-----	-----	-----	-----	-----	-----	-----
Means	+ 2.7	+ 12.3	- 3.4	+ 1.1	- 2.3	+ 0.2	- 8.6

Observers' remarks.

First contacts.—Images quite steady. Sometimes, however, wind shook telescope considerably; (8) doubtful from this cause.—WATSON.

Second contacts.—Not so distinct as in case of transit December 9, 1874.—WATSON. (1) and (2) blurred; (3) (6) (8) good; (4) fair.—NEWCOMB.

Third contacts.—The images were sometimes quite unsteady, and besides the wind shook the observing telescope. The vision was very much worse than in the case of Pekin observation of transit. The uncertainty of these observations greater than in case of transit [of Venus] observations.—WATSON.

Fourth contacts.—Wind shaking telescope. The shadow on the screen began to fall where the contact took place, and interfered with the observations.—WATSON.

SERIES 12. 1876, FEBRUARY 12.

Observer, Prof. GEORGE DAVIDSON, of the United States Coast Survey, and chief of the party stationed at Nagasaki, Japan.

The Sun was, for a few seconds, nearly obscured by clouds at Japan, during which time the first contact took place; and when the fair outline of the solar disk was seen "the edge of the planet was just on the Sun's limb." Professor DAVIDSON, from his study of the painted representations of Venus when ten and twenty seconds on the Sun, was enabled to estimate with some degree of certainty how many seconds Venus had advanced beyond actual contact at the time when he first saw the Sun after the clouds had passed. This estimation he recorded as ten seconds. In observing the artificial transit, Professor DAVIDSON noted the time when Venus appeared to him as he first saw it projected on the solar disk at Nagasaki. Ten seconds were then added to the time of each geometric contact observed by the assistant at the artificial transit, and the resulting instants were then compared with the times noted by Professor DAVIDSON. These comparisons are given in column one, Δt . In the second column, Δt , are given the values from similar estimates of the time when Venus was thought by the observer to be twenty seconds on the Sun. In the second and third columns are given the values of Δt for actual II and III contacts, respectively.

First Contacts + ten seconds.		First Contacts + twenty seconds.		Second Contacts.		Third Contacts.	
No. in order.	Δt s.	No. in order.	Δt s.	No. in order.	Δt s.	No. in order.	Δt s.
1	+ 10.2	1	+ 16.3	1	+ 6.5	1	— 0.5
2	+ 8.7	2	+ 10.6	2	+ 4.8	2	— 7.5
3	+ 12.4	3	+ 17.6	3	+ 1.7	3	— 11.0
4	+ 18.0	4	+ 9.0	4	— 0.7	4	— 3.4
5	+ 8.5	5	+ 13.3	5	— 1.8	5	— 10.8
	—		—	6	— 0.9	6	+ 3.2
Mean	+ 11.6	Mean	+ 13.4	7	— 7.5	7	+ 1.8
				8	— 2.3	8	— 7.3
				9	— 2.3	9	— 5.0
				10	— 1.1		—
				11	+ 2.0	Mean	— 4.5
				12	+ 2.2		
				Mean	00		

Observer's remarks.

First contacts.—Those noted 20^s after were taken because, sooner than that, the objects were too unsteady to form any judgment. All the conditions to-day are more unfavorable, as regards steadiness, than at Nagasaki.—DAVIDSON.

Second contacts.—Atmosphere very unsteady, and wind shaking telescope badly; much more unfavorable than at actual transit.—DAVIDSON.

Third contacts.—Atmosphere very unsteady and images with very diffused edge, whereas it was very steady when I was observing the III contact [at Nagasaki].—DAVIDSON.

SERIES 13 1876, FEBRUARY 12.

The foregoing series of observations was repeated by Professor DAVIDSON, under more favorable atmospheric conditions. The values of Δt in column one are the errors of estimation of Venus ten seconds on the Sun's limb.

First Contacts + ten seconds.		Second Contacts.		Third Contacts.	
No. in order.	Δt s.	No. in order.	Δt s.	No. in order.	Δt s.
1	+ 7.0	1	+ 0.4	1	+ 1.6
2	+ 10.4	2	+ 0.7	2	— 2.1
3	+ 13.0	3	+ 0.9	3	+ 0.7
4	+ 12.7	4	0.0	4	— 0.7
5	+ 15.5	5	+ 0.5	5	+ 1.2
6	+ 12.2	6	+ 1.3	6	+ 2.4
7	+ 12.5	7	+ 1.2	7	— 2.7
8	+ 14.7	8	+ 1.4	8	+ 1.9
	—	9	+ 0.7	9	+ 1.7
Mean	+ 12.2	10	+ 1.0	10	+ 1.6
			—	11	+ 1.1
		Mean	+ 0.7	12	+ 1.5
				Mean	+ 0.7

Observer's remarks.

First contacts.—Objects far better than [in series 12], and very much better than at Nagasaki.—DAVIDSON.

Second contacts.—Atmosphere moderately steady. I think the images *steadier* and *sharper* than at Nagasaki.—DAVIDSON.

Third contacts.—Weather clear. Images *almost* as quiet as at Nagasaki.—DAVIDSON.

Recapitulation of mean results for errors in estimation of contacts.

First Contact.		
	s.	
Observer, HALL . .	+ 3.5	Uncertain from arrangement of screen.
	+ 12.8	Very bad images.
	+ 9.4	Images fair, but not so sharp as the real contact at Wladiwostok.
	+ 7.6	Same remark.
WHEELER	+ 21.1	Not so good as at Wladiwostok.
	+ 17.6	Fair images, but not so well defined as at Wladiwostok.
	+ 8.1	Same remark.

First Contact.		
	^{s.}	
Observer, DAVIDSON	+ 21.6	Estimated 10 ^s after first contact as observed at Nagasaki, but bad seeing.
	+ 22.2	The same, but images good.
WATSON	+ 2.2	Does not correspond to actual transit.
	+ 1.9	Does not correspond to actual transit.
RYAN	+ 9.7	Mean of 7 bad observations. Uncertain.
	+ 7.3	Correspond well to actual observation.

A circumstance to be allowed for in dealing with these results is that the observers improve with practice, and so see the contact sooner. For this reason I am inclined to take for the correction the first one which was derived under conditions corresponding to those of the actual transit. We therefore conclude for the corrections to reduce the actual observations to geometric contact.

Observer, HALL	.	-	^{s.} 9.4
WHEELER		-	17.6
DAVIDSON		-	21.6
RYAN	.	-	7.3

Nothing can be deduced for the correction to WATSON and YOUNG, which must be inferred from the remarks of the observers and the attendant circumstances.

Second Contact.		
	^{s.}	
Observer, HALL	+ 5.5	Images fair.
	+ 3.2	“ “
	+ 2.2	“ “
WHEELER	+ 4.9	Images better than in actual transit.
	+ 4.2	
DAVIDSON	0.0	
	+ 0.7	
WATSON	- 0.4	Not like actual transit.
	- 12.3	“Penumbra.” See observer’s remarks.
	- 3.4	Less disturbed than in actual transit.
YOUNG	+ 6.5	None of the hesitation noted in actual transit.
PETERS	+ 1.1	No remarks.

The following seem to be the most probable corrections to reduce the observations to geometrical contact:

Personal corrections for second contact.

Observer, HALL	-	^{s.} 3.6
WHEELER	-	4.5
DAVIDSON	-	0.4
WATSON	+	5.3
YOUNG	-	6.5
PETERS	-	1.1

		Third Contact.	
		s.	
Observer, HALL	.	—	2.1
WHEELER		—	3.3
DAVIDSON		—	4.5
		+	0.7
WATSON		+	9.0
		+	0.2
YOUNG		—	10.6
RAYMOND		—	1.5

Atmosphere much worse than in actual transit.

Images almost as quiet as at Nagasaki.

See observer's note.

Not like actual transit.

Concluded personal corrections for geometric third contact.

Observer, HALL	.	+	2.1
WHEELER		+	3.3
DAVIDSON			0.0
WATSON		—	6.0
YOUNG		+	10.6
RAYMOND		+	1.5

		Fourth Contact.	
		s.	
Observer, WATSON		—	8.6
YOUNG		—	5.3

Observations interfered with.

The only data for corrections to fourth contact given by the artificial transit are found by changing the signs of these quantities.

§ 3. TABULAR AND OBSERVED TIMES OF CONTACT.

There are two ways in which the comparison with observations of contact may be made; the one by comparing the observed and tabular times; the other by comparing the observed and tabular distances; the observed distance being equal to the sum or difference of the semidiameters of the two bodies, according to whether the contact observed is an external or an internal one. I prefer the latter, but there may sometimes be a difference of opinion what instant of time should be taken as that of the observed contact, and the tabular distance being a function of the time observed cannot be definitively given in this case. The results will therefore be presented in such a form that either mode of comparison may be used.

In the preceding discussion of the photographic observations, formulæ and tables are given whereby the tabular distance and position angle of the centers of the two bodies, as seen from any station, and the coefficients of these quantities for changes in the relative right ascension and declination of the two bodies, and in the horizontal parallax of the Sun, may be readily computed. These have been computed for several times near those of the contacts observed at the several stations, as follows:

Station and Contact.	Greenwich Side- real Time.	Distance of Centers.	$\frac{dS}{dA}$	$\frac{dS}{dD}$	$\frac{dS}{d\pi}$	Position Angle.	$\frac{dp}{dA}$	$\frac{dp}{dD}$	$\frac{dp}{d\pi}$	
Kerguelen I.....	h m s	"				o '				
	7 8	1007.35	+ 0.679	+ 0.676	+ 2.51	47 27.1	+ 128	- 151	- 27	
	7 9	1005.11	+ 0.678	0.679	2.51	47 15.3	129	- 151	- 25	
	7 10	1002.89	+ 0.676	0.681	2.51	47 3.5	129	- 151	- 23	
	7 27 20.2	966.15	+ 0.634	0.725	2.45	43 29.7	143	- 147	+ 13	
	7 29 20.6	962.15	+ 0.629	0.730	2.44	43 4.0	144	- 146	+ 17	
	7 31 20.9	958.21	+ 0.624	0.736	2.44	42 38.0	146	- 146	+ 22	
	7 33 21.2	954.33	+ 0.619	0.741	2.43	42 11.8	148	- 145	+ 26	
	7 35 21.5	950.51	+ 0.614	0.746	2.42	41 45.3	149	- 144	+ 30	
	7 37 21.9	946.72	+ 0.608	0.751	2.41	41 18.7	151	- 144	+ 35	
II.....	7 40	941.92	+ 0.603	+ 0.758	+ 2.40	40 43.4	+ 153	- 143	+ 40	
	7 41	940.09	+ 0.600	0.760	2.39	40 29.9	154	- 143	+ 42	
	7 42	938.29	+ 0.597	0.763	2.39	40 16.3	155	- 142	+ 44	
Campbelltown III..	10 59	939.97	- 0.202	+ 0.976	+ 1.60	- 12 38.5	+ 197	+ 48	- 311	
	11 0	941.89	- 0.205	0.975	1.61	- 12 51.9	197	+ 49	- 310	
	11 1	943.83	- 0.209	0.974	1.62	- 13 5.3	196	+ 49	- 309	
	11 2	945.78	- 0.212	0.973	1.63	- 13 18.7	196	+ 50	- 308	
	11 2 52.4	947.52	- 0.215	0.972	1.64	- 13 30.3	195	+ 51	- 307	
	11 3 45.6	949.26	- 13 42.0	
	11 4 54.8	951.55	- 13 57.2	
	11 5 36.9	952.96	- 14 6.4	
	11 6 55.1	955.58	- 0.231	+ 0.968	1.68	- 14 29.4	193	+ 54	- 302	
Queenstown I.....	6 58	1010.24	+ 0.695	+ 0.658	- 0.26	48 52.3	+ 124	- 154	- 248	
	6 59	1007.89	+ 0.693	0.660	- 0.26	48 40.5	125	- 154	- 249	
	7 0	1005.56	+ 0.690	0.663	- 0.26	48 28.7	125	- 153	- 251	
	7 1	1003.24	+ 0.689	0.666	- 0.25	48 16.8	126	- 153	- 253	
	7 3	998.49	+ 0.684	0.671	- 0.25	47 52.9	128	- 153	- 256	
	7 7	989.44	47 4.3	
	7 11	980.60	46 14.8	
	7 15	971.96	+ 0.657	0.702	- 0.25	45 25.7	137	- 151	- 277	
	7 19	963.51	44 33.3	
	7 23	955.32	43 41.2	
	7 27	947.34	+ 0.626	0.734	- 0.25	42 48.3	147	- 148	- 298	
	II.....	7 28	945.50	+ 0.624	+ 0.736	- 0.25	42 34.9	+ 148	- 148	- 300
		7 29	943.56	+ 0.622	0.739	- 0.26	42 21.5	149	- 147	- 302
		7 30	941.62	+ 0.619	0.742	- 0.26	42 8.0	150	- 147	- 304
	Chatham Island.....	7 16	967.21	+ 0.652	+ 0.707	- 0.05	45 0.7	+ 139	- 151	- 353
7 18		963.03	44 35.0	
7 20		958.93	44 9.1	
7 22		954.88	+ 0.637	0.723	- 0.03	43 42.9	144	- 149	- 366	
7 35		929.90	+ 0.602	0.757	+ 0.02	40 47.6	155	- 145	- 395	
7 36		928.09	40 33.7	
7 37		926.29	+ 0.597	0.762	+ 0.03	40 19.8	156	- 144	- 399	
7 37		926.29	+ 0.597	0.762	+ 0.03	40 19.8	156	- 144	- 399	
Wladiwostok I....	6 54	1008.35	+ 0.721	+ 0.624	- 0.98	51 25.6	+ 118	- 160	+ 483	
	6 55	1005.85	+ 0.720	0.626	- 1.00	51 14.2	+ 119	- 160	+ 482	
	6 56	1003.34	+ 0.718	0.629	- 1.03	51 2.8	+ 119	- 160	+ 482	
	II....	7 21	944.78	+ 0.664	+ 0.695	- 1.40	45 57.1	+ 140	- 157	+ 457
		7 22	942.59	+ 0.661	0.698	- 1.42	45 44.2	+ 141	- 157	+ 455
		7 23	940.43	+ 0.658	0.701	- 1.43	45 31.1	+ 142	- 156	+ 454
7 24		938.28	+ 0.656	0.703	- 1.45	45 18.0	+ 143	- 156	+ 453	

Station and Contact.	Greenwich Side- real Time.	Distance of Centers.	$\frac{dS}{dA}$	$\frac{dS}{dD}$	$\frac{dS}{d\pi}$	Position Angle.	$\frac{dp}{dA}$	$\frac{dp}{dD}$	$\frac{dp}{d\pi}$	
Wladiwostok III....	h m s	"				° '				
	11 14	938.80	- 0.258	+ 0.960	- 1.81	- 16 13.8	+ 195	+ 61	- 432	
	11 15	940.93	- 0.261	0.959	- 1.80	- 16 27.8	194	62	- 434	
	11 16	943.12	- 0.265	0.958	- 1.79	- 16 40.8	193	63	- 435	
Nagasaki I.....	6 55	1008.57	+ 0.719	+ 0.627	- 0.71	+ 50 10.8	+ 118	- 159	+ 460	
	6 56	1006.05	+ 0.716	0.629	- 0.73	50 59.4	119	- 159	+ 459	
	6 57	1003.57	+ 0.714	0.632	- 0.75	50 47.9	120	- 159	+ 459	
	6 58	1001.10	+ 0.713	0.635	- 0.76	50 36.4	121	- 159	+ 458	
	7 14	962.99	+ 0.678	0.677	- 1.01	47 23.6	134	- 158	+ 447	
	7 16	958.44	+ 0.674	0.682	- 1.05	46 58.5	135	- 157	+ 445	
	7 18	953.95	+ 0.669	0.688	- 1.08	46 33.0	137	- 157	+ 443	
	7 20	949.52	+ 0.664	0.693	- 1.10	46 7.4	139	- 157	+ 440	
	II.....	7 22	945.15	+ 0.661	+ 0.698	- 1.13	- 45 41.5	+ 141	- 156	+ 438
		7 23	942.97	+ 0.658	0.701	- 1.15	45 28.5	142	- 156	+ 436
	7 24	940.82	+ 0.655	0.704	- 1.16	45 15.4	143	- 156	+ 435	
	7 25	938.68	+ 0.653	0.707	- 1.18	45 2.2	143	- 156	+ 434	
	7 26	936.54	+ 0.650	0.709	- 1.20	44 49.0	144	- 155	+ 432	
III.....	11 13	939.25	- 0.255	+ 0.961	- 1.53	- 16 3.2	+ 195	+ 61	- 447	
	11 14	941.40	- 0.258	0.960	- 1.52	- 16 16.3	194	+ 61	- 448	
	11 15	943.55	- 0.262	0.959	- 1.51	- 16 30.3	194	+ 62	- 450	
Peking I.....	6 55	1010.51	+ 0.721	+ 0.625	- 0.49	+ 51 19.8	+ 118	- 159	+ 520	
	6 56	1007.99	+ 0.718	0.627	- 0.50	51 8.5	119	- 159	+ 521	
	6 57	1005.51	+ 0.716	0.630	- 0.52	50 57.1	119	- 159	+ 521	
	6 58	1003.04	+ 0.714	0.633	- 0.54	50 45.6	120	- 159	+ 521	
	7 22 54.51	945.07	+ 0.660	0.698	- 0.93	45 41.5	141	- 156	+ 517	
II.....	7 23	944.87	+ 0.661	+ 0.699	- 0.93	+ 45 40.3	+ 141	- 156	+ 516	
	7 24	942.71	+ 0.658	0.702	- 0.95	45 27.3	142	- 156	+ 516	
	7 25	940.55	+ 0.655	0.704	- 0.96	45 14.3	143	- 156	+ 515	
	7 26	938.41	+ 0.653	0.707	- 0.98	45 1.2	143	- 156	+ 514	
III.....	11 15	939.53	- 0.258	+ 0.960	- 1.96	- 16 16.4	+ 195	+ 62	- 356	
	11 16	941.70	- 0.262	0.959	- 1.95	- 16 29.5	194	+ 62	- 358	
	11 17	943.85	- 0.265	0.958	- 1.94	- 16 42.5	193	+ 63	- 361	
	11 19 11.52	948.64	- 0.272	0.955	- 1.92	- 17 10.9	191	+ 64	- 366	
	11 22 19.03	955.57	- 0.282	0.952	- 1.89	- 17 50.8	189	+ 66	- 373	
	11 33 53.53	982.36	- 0.319	0.938	- 1.77	- 20 13.5	182	+ 73	- 395	
IV.....	11 42	1002.10	- 0.343	+ 0.928	- 1.67	- 21 48.9	+ 177	+ 76	- 408	
	11 43	1004.58	- 0.346	0.927	- 1.66	- 22 0.3	175	+ 77	- 409	
	11 44	1007.08	- 0.348	0.926	- 1.65	- 22 11.7	175	+ 77	- 410	

The assumed semidiameters for mean distance unity are:

$$\begin{aligned} \text{The Sun, } \sigma_0 &= 960''.00 \\ \text{Venus, } \sigma'_0 &= 8''.50 \end{aligned}$$

These values give for the geocentric semidiameters during the time of the transit:

$$\begin{aligned} \text{The Sun, Ingress, } &974''.974 + \delta \sigma \\ &\text{Egress, } 974''.972 + \delta \sigma \\ \text{Venus - - - - - } &32''.156 + \delta \sigma' \end{aligned}$$

The corrections to reduce these last values to those of AIRY and HILL, respectively, are:

$$\begin{aligned} \text{The Sun, } \delta \sigma \text{ (AIRY)} &= + 1''.850 & \delta \sigma \text{ (HILL)} &= - 0''.212 \\ \text{Venus, } \delta \sigma' \text{ (AIRY)} &= - 0''.736 & \delta \sigma' \text{ (HILL)} &= + 0''.174 \end{aligned}$$

The corrections to reduce the relative right ascension and declination of Venus to AIRY'S values are:

$$\begin{aligned} \text{Ingress, } \delta A &= - 3''.62; \delta D = - 0''.95 \\ \text{Egress, } \delta A &= - 3''.54; \delta D = - 0''.98 \end{aligned}$$

The quantity by which the assumed constant of solar parallax, $8''.848$, must be corrected to reduce it to that assumed by AIRY, $8''.95$, is

$$\delta \pi = + 0''.102$$

We now have from the preceding data the following summary of the tabular and observed times of contact, which must be examined in connection with the observations to appear in Part II. The first column, which needs explanation, is that giving the Greenwich sidereal times of observation. Here, $\delta \lambda_1$, $\delta \lambda_2$, etc., indicate the corrections to the assumed west longitudes of the stations, which have been already given. The reductions to geometric contacts have been already given, in part, as deductions from the observations with the artificial transit. In some cases, however, the observer either made no observations with this instrument, or those he did make are of doubtful applicability. In these cases, the most probable reduction is judged of from other circumstances, or, when there are no data for forming a judgment, the column is filled with a mark of interrogation. The following are the cases in which the reduction does not depend solely on observations with the artificial transit.

Nagasaki, III. DAVIDSON.—The Sun being obscured for a few seconds at the moment of contact, the observer judged contact to have passed 5 seconds when it reappeared. Hence, reduction = $- 5^s$.

Peking, I. WATSON AND YOUNG.—The reductions are inferred from the remarks of the observers respecting the visibility of the planet at intervals of a few seconds after the recorded times, compared with the general character of the results with the artificial transit. It will be noted that YOUNG was not fully certain that he saw the planet until 10 seconds after the recorded time; the correction is therefore smaller than it would otherwise have been.

Queenstown, I. PETERS.—For the correction the usual time after geometric contact at which the indentation becomes plainly perceptible is taken.

Summary of times of contact noted at the several stations, with the concluded reductions to geometric contact, as derived from observations of the artificial transit.

No. for Ref.	Station.	Contact.	Observer.	Greenwich Sid. Time of Observation.	Red. to Geo- metric Phase.	Green. Sid. Time of Geometric Phase.	Remarks.
1	Wladiwostok . . .	I	HALL	h m s 6 55 51.2 + $\delta\lambda_1$	— 9.4	h m s 6 55 41.8 + $\delta\lambda_1$	Very doubtful.
			WHEELER	6 55 56.5 “	— 17.6	6 55 38.9 “	
2		II	HALL	7 23 12.6 “	— 3.6	7 23 9.0 “	
			WHEELER	7 23 19.0 “	— 4.5	7 23 14.5 “	
3		III	WHEELER	11 15 42.9 : :	+ 3.3	15 46.2 : :	
4	Nagasaki	I	DAVIDSON	6 57 9.7 + $\delta\lambda_2$	— 21.6	6 56 48.1 + $\delta\lambda_2$	
			TITTMANN	6 57 24.3 “	(?)	
5		II	DAVIDSON	7 25 4.2 “	— 0.4	7 25 3.8 + $\delta\lambda_2$	
			TITTMANN	7 23 31.6 “	(?)	7 23 31.6 “	
6		III	DAVIDSON	11 14 9.7 “	— 5 (?)	11 14 4.7 “	
			TITTMANN	11 14 29.5 “	(?)	11 14 29.5 “	
7	Peking	I	WATSON	6 57 28.5 + $\delta\lambda_3$	— 7.0	6 57 21.5 + $\delta\lambda_3$	
			YOUNG	6 57 23.5 “	— 5.0	6 57 18.5	
8		II	WATSON	7 25 28.3 “	+ 5.3 (?)	7 25 33.6 (?)	
			YOUNG	7 25 1.0 “	— 6.5	7 24 54.5	
9		III	WATSON	11 15 32.2 “	— 6.0	11 15 26.2	
			YOUNG	11 15 13.3 “	+ 10.6	11 15 23.9	
			WOODWARD	11 15 31.3 “	(?)	11 15 31.3	
10		IV	WATSON	11 42 50.3 “	+ 8.6	11 42 58.9	
			YOUNG	11 42 33.7 “	+ 10.:	11 42 43.7	
			WOODWARD	11 42 24.7 “	+ 20.2	11 42 44.7 :	
11	Kerguelen	I	RYAN	7 9 35.1 + $\delta\lambda_4$	— 7.3	7 9 27.8 + $\delta\lambda_4$	
12	Campbelltown . . .	III	RAYMOND	11 0 22.0 + $\delta\lambda_6$	+ 1.5	11 0 23.5 + $\delta\lambda_6$	
13	Queenstown	I	PETERS	7 0 3.5 + $\delta\lambda_7$	— 15. (?)	6 59 48.5 + $\delta\lambda_7$	
14		II	PETERS	7 29 51.5 “	— 1.1	7 29 50.4 “	

Greenwich sidereal times of tabular contact for comparison with the above observations.

No. for Ref.	h	m	s	s	s	
1.	6	54	29.5	+ 17.3	δA	+ 15.0 δD - 24.0 ($\delta \sigma$ + $\delta \sigma'$) - 23 $\delta \pi$
2.	7	21	54.5	+ 18.2	δA	+ 19.2 δD - 27.5 ($\delta \sigma$ - $\delta \sigma'$) - 39 $\delta \pi$
3.	11	15	52.1	+ 7.2	δA	- 26.2 δD + 27.4 ($\delta \sigma$ - $\delta \sigma'$) + 49 $\delta \pi$
4.	6	55	34.5	+ 17.2	δA	+ 15.1 δD - 24.0 ($\delta \sigma$ + $\delta \sigma'$) - 17 $\delta \pi$
5.	7	23	4.8	+ 18.1	δA	+ 19.3 δD - 27.5 ($\delta \sigma$ - $\delta \sigma'$) - 32 $\delta \pi$
6.	11	14	39.5	+ 7.2	δA	- 26.4 δD + 27.5 ($\delta \sigma$ - $\delta \sigma'$) + 42 $\delta \pi$
7.	6	56	21.2	+ 17.2	δA	+ 15.1 δD - 24.0 ($\delta \sigma$ + $\delta \sigma'$) - 12 $\delta \pi$
8.	7	23	57.5	+ 18.2	δA	+ 19.4 δD - 27.7 ($\delta \sigma$ - $\delta \sigma'$) - 26 $\delta \pi$
9.	11	16	31.3	+ 7.3	δA	- 26.5 δD + 27.7 ($\delta \sigma$ - $\delta \sigma'$) + 54 $\delta \pi$
10.	11	44	1.2	+ 8.4	δA	- 22.2 δD + 24.0 ($\delta \sigma$ + $\delta \sigma'$) + 40 $\delta \pi$
11.	7	8	6.2	+ 18.1	δA	+ 18.0 δD - 26.7 ($\delta \sigma$ + $\delta \sigma'$) + 67 $\delta \pi$
12.	11	0	28.8	+ 6.2	δA	- 30.1 δD + 31.1 ($\delta \sigma$ - $\delta \sigma'$) - 51 $\delta \pi$
13.	6	59	19.3	+ 17.8	δA	+ 17.0 δD - 25.7 ($\delta \sigma$ + $\delta \sigma'$) - 7 $\delta \pi$
14.	7	29	22.9	+ 19.2	δA	+ 22.8 δD - 30.9 ($\delta \sigma$ - $\delta \sigma'$) - 8 $\delta \pi$

After reading Professor WATSON'S report of his observations, the writer was led to doubt the strict applicability of the corrections for internal contact derived from the artificial transit, partly from the want of correspondence between the actual observations and those made on the model, and partly because many observers did not determine their errors, and it is desirable that all the observations should be discussed on a uniform plan. This is a question to be considered by such persons as discuss all the observations.*

We next give a similar comparison with respect to distance of centers instead of times. The first column of the following table gives the name of the observer, the contact observed, and the time actually noted, reduced to Greenwich sidereal time, as before. The sum of all the corrections to each of these times, including correction of longitude, reduction to geometric phase, and errors of observation, is represented by the general symbol τ . This quantity has a separate and independent value for each observation. The determination of its most probable value and probable error is left for the final discussion of the observations.

* The present discussion was prepared during the years 1876 and 1877, and is printed without alteration. Since that time several publications of contact observations have been made, the study of which leads me to doubt the advisability of applying the above corrections derived from the artificial transit to the observed times of internal contact.

1. It is necessary that all the observations shall be reduced on a uniform plan, and the personal corrections derived by the artificial transit are not known to be satisfactorily attainable for any but the American parties, and only for a small part of them.

2. There are various nonconformities between the observations of the real and those of the artificial transit, which have been pointed out in the preceding discussion.

It therefore appears likely that the best way of treating the observations will be to infer the most probable times of contact from the descriptions and drawings of the observers.

These conclusions do not detract from the value of the artificial transit as a method of previous practice on the part of the observers. Moreover, if most of the observers could be practiced on a uniform system, and with an instrument showing phenomena as much as possible like those of the real transit, I think the corrections might be regarded as real.—THE EDITOR.

Next, the observed distance of centers is given, being supposed equal to the sum or difference of the semidiameters. The symbolic corrections of the semidiameters are here included.

Thirdly, the tabular distance of centers as affected with parallax is given. It is interpolated from the values near the times of contact given on pages 135 and 136.

Observer, Contact, and Greenwich Sidereal Times.			Observed Distance of Centers.	Tabular Distance of Centers.
		h m s	"	" "
HALL.....	I	6 55 51.2 + τ	1007.12 + $\delta\sigma + \delta\sigma'$	1003.71 - .042 τ + 0.72 δA + 0.63 δD - 1.02 $\delta\pi$
WHEELER ..	I	6 55 56.5 + τ	1007.12 "	1003.48 - .042 + 0.72 + 0.63 - 1.03
HALL.....	II	7 23 12.6 + τ	942.80 + $\delta\sigma - \delta\sigma'$	939.99 - .036 + 0.66 + 0.70 - 1.44
WHEELER ..	II	7 23 19.0 + τ	942.80 "	939.76 - .036 + 0.66 + 0.70 - 1.44
WHEELER ..	III	11 15 42.9 + τ	942.80 "	942.49 + .037 τ - 0.26 + 0.96 - 1.79
DAVIDSON ..	I	6 57 9.7 + τ	1007.12 + $\delta\sigma + \delta\sigma'$	1003.18 - .042 + 0.71 + 0.63 - 0.75
TITTMANN ..	I	6 57 24.3 + τ	1007.12 "	1002.57 - .042 + 0.71 + 0.63 - 0.75
DAVIDSON ..	II	7 25 4.2 + τ	942.80 + $\delta\sigma - \delta\sigma'$	938.53 - .036 + 0.65 + 0.71 - 1.18
TITTMANN ..	II	7 23 31.6 + τ	942.80 "	941.83 - .036 + 0.66 + 0.70 - 1.16
DAVIDSON ..	III	11 14 9.7 + τ	942.82 "	941.75 + .036 - 0.26 + 0.96 - 1.52
TITTMANN ..	III	11 14 29.5 + τ	942.82 "	942.46 + .036 - 0.26 + 0.96 - 1.52
WATSON	I	6 57 28.5 + τ	1007.12 + $\delta\sigma + \delta\sigma'$	1004.34 - .041 + 0.72 + 0.63 - 0.52
YOUNG	I	6 57 23.5 + τ	1007.12 "	1004.54 - .041 + 0.72 + 0.63 - 0.52
WATSON	II	7 25 28.3 + τ	942.80 + $\delta\sigma - \delta\sigma'$	939.55 - .036 + 0.65 + 0.70 - 0.97
YOUNG	II	7 25 1.0 + τ	942.80 "	940.51 - .036 + 0.65 + 0.70 - 0.96
WATSON	III	11 15 32.2 + τ	942.82 "	940.71 + .036 - 0.26 + 0.96 - 1.96
YOUNG	III	11 15 13.3 + τ	942.82 "	940.01 + .036 - 0.26 + 0.96 - 1.96
WOODWARD.	III	11 15 31.3 + τ	942.82 "	940.67 + .036 - 0.26 + 0.96 - 1.96
WATSON	IV	11 42 50.3 + τ	1007.14 + $\delta\sigma + \delta\sigma'$	1004.18 + .041 - 0.35 + 0.93 - 1.67
YOUNG	IV	11 42 33.7 + τ	1007.14 "	1003.50 + .041 - 0.34 + 0.93 - 1.67
WOODWARD.	IV	11 42 24.7 + τ	1007.14 "	1003.13 + .041 - 0.34 + 0.93 - 1.66
RYAN.....	I	7 9 35.1 + τ	1007.12 "	1003.80 - .037 + 0.68 + 0.68 + 2.51
RAYMOND...	III	11 0 22.0 + τ	942.84 + $\delta\sigma - \delta\sigma'$	942.60 + .032 - 0.71 + 0.97 + 1.61
PETERS	I	7 0 3.5 + τ	1007.14 + $\delta\sigma + \delta\sigma'$	1005.42 - .039 + 0.69 + 0.66 - 0.26
PETERS	II	7 29 51.5 + τ	942.82 + $\delta\sigma - \delta\sigma'$	941.89 - .039 + 0.62 + 0.74 - 0.26

APPENDIX I.*

COAST SURVEY OFFICE,

Washington, D. C., January 2, 1874.

DEAR SIR: Upon the subject of Atmospheric Dispersion, as connected with photographs of the Transit of Venus, I take the liberty to send you the inclosed notes as giving my own view of some of the questions involved.

Yours, truly,

J. HOMER LANE.

Professor NEWCOMB.

CORRECTION OF ATMOSPHERIC DISPERSION IN ASTRONOMICAL PHOTOGRAPHY.

The design of this paper is to make some suggestions in regard to the correction of atmospheric dispersion in taking photographs of the approaching Transit of Venus. So far as I have heard, no attention has been drawn to the possible influence of atmospheric dispersion upon a parallax deduced from photographs.

There are two ways in which the atmospheric dispersion may cause a displacement of the assigned position of the photographic print of the limb of Venus relatively to the photographic print of the Sun's limb, and any such displacement enters, of course, with its full amount directly into the deduced parallax of Venus from the Sun.

In the first place, since the radiation which reaches us from the Sun's limb is less intense than that of the disc within, the atmospheric spectrum of a radiant point must be expected to leave in the photograph a longer print for the limb of Venus than for that of the Sun, and the excess may fall more towards one end of the spectrum than towards the other.

In the second place, the absorption of the Sun's atmosphere, to which the inferior brightness of the Sun's limb is due, may, for the photographic rays, be greater towards one end of the spectrum than towards the other.

Although the amount of this displacement will doubtless be small, being, of course, less than the length of the air spectrum of photographic rays, yet it must be remembered that at a zenith distance of 45° the whole refraction will be something like 4 times greater than the corresponding parallax of Venus from the Sun; and if the method be applied at a greater zenith distance, the ratio will be still greater. On my mentioning this subject to Prof. C. S. PEIRCE, he referred me to some investigations of KETTELER on the chromatic dispersion of air and several gases. KETTELER'S

* The very able author of this note died while it was in the printer's hands, and therefore had no opportunity to revise the proof.—S. N.

experiments were made with the monochromatic lights of the lithium flame, the sodium flame, and the thallium flame; and his results are confined to the three corresponding points of the spectrum. At a zenith distance of 45° , his numbers give, for the lithium and thallium lines—wave lengths $0^{\text{mm}}.00067061$ and $0^{\text{mm}}.00053451$ —an atmospheric dispersion of about $0''.9$. I take this space for comparison, supposing it to be, as I find it in a curve given by ROSCÖE, equal to the space which includes the denser parts of the chemical spectrum.

I have been informed by Professor LYMAN that Professor AIRY has introduced a correction of atmospheric dispersion into the eye-piece. For photographing the transit it would of course be imperative to introduce the correction at the object-glass; and the way of all others to do it will be by means of an eccentricity of the crown and flint lenses of the object-glass, made to vary with the tangent of the zenith distance.

Now, as the zero from which to measure this eccentric motion of the one lens over the other depends on the perfect centering of the two lenses, the suggestion becomes an obvious one that the whole object-glass may be rotated 180° for alternate photographs taken near together in order to extinguish any error of the centering or zero; and this half rotation of the object-glass would be equally useful even if any other method were chosen for extinguishing the atmospheric spectrum.

Let

f_1 and f_2 represent the stellar focal lengths of the *one* lens that is moved eccentrically over the *aperture* for two assumed points of the photographic spectrum.

n_1 and n_2 , the corresponding indices of refraction of the glass composing it.

v_1 and v_2 , the corresponding indices of refraction of the air.

z , the zenith distance;

and e , the eccentricity of the lens in linear measure of the same units as

f_1 and f_2 .

Then the proper value of this eccentricity is

$$e = f_1 f_2 \frac{v_2 - v_1}{f_1 - f_2} \tan. z,$$

or

$$c = f_1 (n_1 - 1) \frac{v_2 - v_1}{n_2 - n_1} \tan. z.$$

It seems highly improbable that the irrationality of the air spectrum with the glass spectrum—in the photographic rays, where, I believe, it has not yet been determined—should prove to be so great as to defeat the complete extinction, practically, of the atmospheric spectrum in this way.

Supposing the inner surface of the flint lens of the objective to be flat, an eccentric displacement of this lens, sliding on its concave surface, is equivalent to the introduction of a refracting prism in the manner of the compound prism of BOSCOVICH; and the fact that this prism is introduced into the converging rays at the back of the objective, instead of being introduced, as an extra prism would be, into the parallel rays in front, gives the measure of the effect which the eccentricity produces upon

spherical aberration. Evidently, this effect is altogether insignificant in comparison with the dispersion produced or corrected, the dispersion for the lines D and G being, in flint glass, say $\frac{1}{26}$ th the refraction.

The question arises whether the two glasses of the objective might have individual mutually-compensated irregularities of form, of which the compensation would be disturbed by the eccentric displacement. I do not suppose this likely, and trial would probably show the dispersal effect to stand sensibly alone.

Besides, an effect depending on the supposed irregularities of the lenses can hardly be supposed to have any common law of connection, in different instruments, with the direction of the displacement and the parallax. The atmospheric spectrum has not, so far as I know, been successfully examined in the more refrangible part, and I do not know as we could do better at present than to assume the whole air spectrum to be similar to the glass spectrum.

On this assumption, applying the above formula to a crown-glass lens with indices of refraction given by FRAUNHOFER for a specimen of crown glass, we have

$$\frac{v_2 - v_1}{n_2 - n_1} = \frac{1}{3100}, \text{ and } e = \frac{0.526}{3100} f_1 \tan z = 0.000169 f_1 \tan z.$$

I need not repeat that f_1 here means the focal distance of the lens alone, not of the objective.

APPENDIX II.

RECORDS OF CONTACT OBSERVATIONS.

When the present publication was sent to press it was intended that Part II, containing the observations made at each station, should follow it immediately. But some of the reductions are still unfinished, and it is deemed advisable to await their completion rather than to issue the observations in an imperfect form. As it is desirable that the observations of contact should be available to investigators without further delay, they are here presented in full, as copied from original notes of the observers. No discussion has been attempted, for the reason that this can be most profitably undertaken in some work including all the available observations of the transit. The time reductions are, however, applied to the chronometer observations, and may be regarded as definitive, with the exception of the longitudes, some of which require further investigation.

WLADIWOSTOK.

Professor Hall's notes of observations of contact.

	h.	m.	s.		
NEGUS 772.	{	1	44	43.5	— 8 ^s .5*
First contact of Venus.		1	44	35.0	
NEGUS 772.	{	2	10	20.5	—Black drop begins.
Second contact.		2	10	52.0	—Contact ?
		2	11	29.5	—Planet wholly on.

At second contact planet faint on account of haze, and observation difficult.

Times of the above contacts were noted by Mr. GARDNER on chronometer NEGUS 772; also the second contact by Lieutenant MORONG.

Third contact lost by clouds. Strained my eye very much, but could not be sure of the third contact.

The preceding observations were made with the ALVAN CLARK telescope 856, and with the plain eye-piece and power of 140. I tried to use the double-image micrometer, but the haze was too dense. At first contact the sun's limb very steady and well defined, and I was looking near the right point. At second contact the haze was much more dense, but think I saw the real contact. At first the contact between the planet and Sun's limb seemed to lengthen, and this is the time first noted. Then flashes of light passed between the planet and Sun's limb, and this is the time noted as the contact. There is some doubt about the minute of second contact, which may be 11, as the hour and minute hands were near each other.

When the haze gave us a view of the Sun, Venus was seen beautifully defined; she was much sharper in outline than I expected. I think that without the haze the contacts could have been very exactly observed.

A. HALL.

P. S.—I had been looking at the planet very steadily for three or four minutes, and my eye was tired and strained a little, I think, at second contact.

Have never seen the Sun's limb steadier or better defined than at first contact, and five or six minutes before second contact.

* This 8^s.5 is the time counted after seeing contact before noting chronometer, which was done at 1^h 44^m 43^s.5. The second time is therefore that at which the contact was observed.

At third contact a good prospect of getting a good observation five minutes beforehand, but haze rapidly grew dense and the Sun quite low. Think I should have got this contact but for the colored shade, which I did not dare to take off. The same power used in all the contact observations. It was the plain eye-piece of power 140.

Mr. O. B. Wheeler's notes.

ALVAN CLARK 3-inch objective equatorially-mounted telescope. Power, 30.62.

	h.	m.	s.
Contacts.—First:	10	30	47.0, good.
Second:	10	58	5.0, poor; too cloudy.
Third:	2	49	50.5, very poor!! not at all reliable; too cloudy.

Remarks.—Very good image through haze at first contact, but could not use the higher powers of eye-pieces.

Clouds getting thicker, and second contact observed with great difficulty. .

“Black drop” not definitely seen.

Very favorable about ten minutes before third contact.

Disk of Sun not visible at time of fourth contact.

Spots on the Sun seen very finely once, and only once, during the day, at about 2^h 20^m p. m.

O. B. W.

NAGASAKI.

Prof. George Davidson's notes of his observations.

DECEMBER, 9 a. m., civil, 1874.

Meridian instrument U. S. C. S. 2 leveled and yet on the meridian mark. Chronograph in working order. At 9 a. m. lower stratum of clouds partially broke away. Sky covered with cirrus. Took reversed pictures in photograph work.

Bank of heavy clouds forming to southward; cirrus and cirro-stratus above. Equatorial clock running well. At 10 a. m. clouds are thicker; at 10^h 13^m, mean time, slight break in clouds. Commenced watching at English computed time; clouds off and on.

I. Contact by chronometer 1563 (sidereal break circuit), 15^h 36^m 52^s.

Sun nearly obscured. Observation doubtful; Venus on. I judge time within ten seconds. Passing clouds nearly obscured the Sun after coming on of planet. The Sun just before the time unsteady.—[Mrs. G. D., Recorder.]

At 15^h 42^m the Sun was obscured by clouds and continued so. The lightest colored shade was used during the first contact; image white.

II. Contact, 16^h 4^m 46^s.5.—[Observer, G. D.; Recorder, G. D.]

Sun partially obscured and image faint, but have no doubt about it within two seconds.

[There was no ligament or black drop, but there was a slight atmospheric disturbance by which the exact instant of the joining of the cusps could not be determined with the precision which I noted in the total eclipse of 1869. But the disturbance was exactly what I have been accustomed to for the last twenty-nine years in the geodetic and astronomical work of the Coast Survey. I noted the time when the dark disturbed overlapping of the limb of Venus over the limb of the Sun was discontinued.]

This note added December 11. I was down with neuralgia all the 10th.

III. Contact, chronometer 1563, 19^h 53^m 52^s. This is a few seconds past contact; no colored glass; flying thick clouds. Ten or fifteen seconds before, I saw the clear separation; no ligament or band; clouds covered it at exact instant and opened just after, but I feel that it was not past 5 seconds. With sharp outline; then clouds.

December 9, 1874.—Compared 1503 and 1563 on the chronograph sheet and by coincidence:

	h.	m.	s.		h.	m.	s.	
1742 =			10.0		1742 =	3	39	52.0

December 9, 1874.—Comparison chronometers:

	h. m. s.		h. m. s.
1742 =	6 43 32.0		1742 = 6 44 52.0
1563 =	23 55 43.5		1503 = 23 56 48.5

Rain during night and heavy gusts of wind.

Mem. from Tittmann's book.—Time of second contact, 10^h 52^m 20^s. He was using chronometer 1742: Hassler 3-inch equatorial of the Coast Survey. On December 12 he informed me that when he had taken his eye from the telescope to note the "tens," he heard Captain YANAGI call out "time" to his assistant, thus indicating that he had observed it 4 seconds later than TITTMANN. YANAGI was using U. S. C. S. Recon. telescope No. 35, 1½-inch objective.—G. D., December 16, 1874.

DECEMBER 11, 1874.

I was too ill yesterday to give a *résumé* of what was done on the 9th. Everything was in good working order. The equatorial was running well; the chronograph ditto. Photographic arrangements complete and satisfactory; heliostat doing well. In the photograph-room were Messrs. SEIBERT, LODGE, WILLIAMS, and AYENS, photographers, and Professor MURRAY recording. In adjacent room were photographers assisting in

One hour before the transit I had made the measurements for focal distance of the photographic objective, and with Mr. TITTMANN had examined all the adjustments and found them as per record. And here it is proper to state that from the beginning no change has been necessary in said adjustments.

Reversed pictures had been taken, and all were waiting. Transit observations had been taken about 4.30 a. m., when clouds suddenly covered the sky. The clouds began to thicken at 9.30 a. m. There were two strata, the upper of cirrus and cirro-stratus forming a complete covering over the whole sky; the lower was a stratum of heavy cumulo-stratus, forming nearly as low as 2,000 feet (it nearly capped the mountain 1,900 feet high and 4 miles south of us). This lower stratum came from the southward. There was little wind at the station, but the clouds flying across the sky were from the SW.

At 10 a. m. the prospect looked very bad; Sun not seen through the clouds. At 10.13 whilst at the equatorial the clouds of lower stratum began to break in the vicinity of the sun, but thick below it. At 15^h 33½^m, by chronometer 1563, began to watch steadily for the first contact; clouds flying over sun; upper stratum still existing as an upper curtain.

Changed colored glass to lightest shade before commencement of watching. Sometimes Sun nearly clear, then almost instantly obscured, again partially out.

The Sun was nearly obscured at time of contact, and when light permitted fair outline to be seen the edge of planet was just on the Sun's limb at the exact point at which I had all along been looking.

From my practice on the artificial Venus, I judge the planet was ten seconds on at the time I noted.

About 7 or 8 seconds thereafter I called out to the photographers to commence, and I prepared to obtain measures of the planet on the Sun as I had in the artificial Venus, but it was too much obscured by clouds to admit such measures, and no sharpness of outline. [(a) I should have stated that here as well as at I. contact I tried to observe without any shade, but when the Sun would suddenly burst through, it was too much for my eyes, and this record, therefore, refers to the conditions when I had the shade on.—G. D., December 16, 1874.]

I had then to wait for better light, and when the planet was half on the limb of the Sun I began measures of cusps. The Sun's limb was quite unsteady just before first contact, and I could not see any approach of Venus nor any different indications at point of contact from other parts of the limb to which I occasionally looked.

The measures of cusps were made with varying conditions of the Sun's brightness, but from my practice on the artificial Venus I was prepared for the work. The cusps were not so bright as I could desire. They were made with lightest shade. Without shade I could not make them. A shade between them would have been of great help.

After cusps I turned micrometer to zero for the second contact. This I got as well as such an observation can be made, and I am satisfied that it has not an error of 2^s.

There was no black drop, no ligament, and only a slight disturbance of the limbs that prevented a sharp separation such as I had in the total solar eclipse of 1869 in Alaska.

But this disturbance was what I have been accustomed to meet with in twenty-nine years' experience on the Coast Survey at all moderate elevations.

After the second contact I turned micrometer and commenced measures for the separation of the limbs, and continued them until Venus was about one diameter from the Sun's limb. The position of the micrometer was not changed during these measures.

It then clouded up and prevented further observations. Before noon it broke away slightly, and then I observed the transit of the limbs of the Sun and of Venus over the nine threads of the United States Coast Survey meridian transit No. 2; recording same on fillet of Coast Survey chronograph, my son watching the running of the fillet, &c.

Mr. TITTMANN observed the difference of declination of the upper limb of the Sun with the upper and lower limbs of Venus in the STACKPOLE transit instrument 1507, at and near the meridian passage.

In my meridian transit observations I used two glasses from my sextant, giving a light orange image of the Sun. The planet was moderately sharply defined after the first three threads, and until the last one. The Sun's limbs were moderately sharply defined. The transits are not as good as star transits. The Sun continuing slightly unobscured, I returned to the equatorial and observed the horizontal diameter of Venus with the double-image micrometer, discontinuing observations when clouds overshadowed us.

In subsequent openings Mr. TITTMANN and Professor MURRAY made some measures, the latter as showing what the instrument would do in the hands of an inexperienced observer. Clouds covered the Sun densely, and Mr. EDWARDS had no opportunity for making measures.

The Sun was wholly obscured until Venus was about one diameter from the western limb, when the clouds broke for two or three minutes and I was commencing diameter measures, but clouds again hindered. I then watched without colored glass until near third contact, the Sun showing occasionally, but changing from bright to obscure rapidly, quicker than I could change shades. Near third contact the seeing was good without shade until 10 or 15 seconds of the contact, during which time the line of separation was well defined and moderately sharp. No sign of ligament, black drop, or distortion. If anything, it promised to be better than second contact, but just as the line of separation was closing, a dense cloud drifted over the Sun for a few seconds, and when it had passed the line of separation was just past: I should judge about five seconds; not more, and probably less.

Then clouds drifted over in a dense bank, and we saw no more Sun. At 4 p. m. the rain began to fall and the night was threatening; but at dark a slight break was noticed in the clouds, and I immediately went to transit No. 2 and managed to observe the stars, as per record.

As the photographic arrangement was complete, and Mrs. DAVIDSON was in the equatorial observatory as recorder, and my son to hold the chronometer quite near my ear (to prevent interference by the beat of the equatorial pendulum), I gave Mr. TITTMANN the use of the Coast Survey equatorial of 3 inches, using astronomical eye-piece and power of —.

Hereto he adds his record of what he saw therewith:

My account is contained in the transit book used on the occasion to note the times of contact in. The notes were made immediately after each contact, and I have nothing to add save that the expression there used, viz, "the disturbance of the Sun's limb * * * not nearly so much disturbance as during the Washington artificial transits at their *worst*," is not meant to imply a great disturbance.—O. H. T., December 14.

There was no part of the time during the transit when the Sun was seen from a clear, blue sky. During the whole time of transit there was that upper stratum of moderately thin clouds that would have been a decided benefit, as they prevented the heat rays coming through to the lower atmosphere and creating irregular undulations of the atmosphere. As far as Volcano Wunxen, 30 miles eastward, the outline of objects was sharp and steady; the same to the north; to the southward were heavy clouds of cumulo-stratus and threatening weather. The atmosphere was mod-

erately steady throughout, at least so we would designate it in Coast Survey work. Had there been no upper stratum of clouds and only the lower stratum, I am satisfied from experience elsewhere and here that the atmospheric disturbances would have been much greater.

I have other suggestions to make upon the questions which arise as to size of telescope, details in instruments, and the necessity for high elevations for such work; but I reserve them for my report to the President of the Commission.—D.

Mr. Tittmann's notes of his observations.

DECEMBER 9.

Transit of Venus, $10^{\text{h}} 26^{\text{m}} 17^{\text{s}}$ mean time of first contact noted by mean time chronometer 1742. It was not seen until after Professor DAVIDSON called time to the photographers. It was then seen further on the Sun than twice the amount of the artificial Venus on the hill through the same telescope. $10^{\text{h}} 52^{\text{m}} 20^{\text{s}}$ time of second internal contact. The Sun seen through haze and not much apparent disturbance of the Sun's limb. Not nearly so much disturbance as during the Washington artificial transits at their worst. The line of light broke clearly and in an apparent true continuation of Venus's limb, and when expected.

Third internal contact, mean time chronometer 1742, $2^{\text{h}} 42^{\text{m}} 40^{\text{s}}$. Sun seen through break in the clouds and viewed without a shade; clouds passing interfered with the exactness of the determination, but apparently the clouds caused the only uncertainty.

Observer cannot form estimate of the error of his notation.

Instrument used U. S. C. S. equatorial No. 12; aperture, 3 inches; focal length, $46''\cdot 5$; made by UTSCHNEIDER & FRAUENHOFER, Munich.

PEKIN.

Professor Watson's notes.

WEDNESDAY, December 9, 1874

Transit of Venus. Micrometer set at $16^{\circ}\cdot 204$.

First contact $1^{\text{h}} 55^{\text{m}} 10^{\circ}\cdot 0$; uncertain; Sun shining through clouds, and could scarcely see well with shade on, and too bright to observe without it.

Internal contact $2^{\text{h}} 22^{\text{m}} 5^{\circ}\cdot 0$. This time uncertain; could not see the planet well steadily; sometimes only glimpses of it; it is probably 5 seconds and possibly 10 seconds late; no black ligament; faint gleams of light across 30° earlier.

At first contact the planet just clearly indented the Sun's limb, as I could faintly see it. Time uncertain, 3 or 4 seconds. At internal contact had sometimes only glimpses of Venus. When time recorded a momentary good glimpse showed the planet just fairly free from Sun's limb; could not see any black ligament; definition good; observed line of white light completed. I am not sure that the minute is correct for first contact. The seconds of contact were instantly recorded and the minute afterwards. I do not think the time seconds are more than 3 or 4 seconds out, and of course late.

Third contact at $6^{\text{h}} 11^{\text{m}} 30^{\circ}\cdot 5$.

At $6^{\text{h}} 11^{\text{m}} 6^{\circ}\cdot 0$ band of light momentarily disappeared, and I supposed at first it was the contact, but the line was immediately re-established and remained so until the time recorded as contact.

The time of third contact is good; not uncertain half a second. Just as chronometer ticked $30^{\circ}\cdot 5$ the planet and sky were instantly united. The instant appeared as sharply defined as the immersion of a star at the moon's limb in an occultation. For about 15° after the contact the space between the cusps was faintly illuminated with an even tint of light. Before the contact the line of light was disturbed by irregular shadows.

Last contact at $6^{\text{h}} 38^{\text{m}} 44^{\circ}\cdot 0$; time by BOND 290. All the times of the contacts and measures are from BOND 290.

The limb of the Sun was very unsteady, and after the last measures the cusps became so blunt and ill defined that I ceased measuring to be ready for last contact. I think the time observed is not more than a second out. I could see the planet certainly to the very edge of the Sun, although edge undulating very much. At $43^{\circ}\cdot 0$ I was sure I could see it, and at $45^{\circ}\cdot 0$ I could not see it. I therefore estimated the time at $44^{\circ}\cdot 0$, at which time I feel sure the planet was clear of the Sun's limb.

WEDNESDAY, *December 9, 1874.*

The notes accompanying the foregoing observations were made at the time of the observations as soon as an opportunity afforded after they were taken. In order to make the record intelligible, I record here, a few hours later, while the phenomena witnessed are fresh in my mind, a circumstantial statement of my observations with the equatorial. At 9^h a. m., local time, I put the equatorial on the Sun, which could be just discerned through the clouds. The preceding night had been clear and the Sun rose in clear sky, but at 8 o'clock a. m. it began to cloud up. As the time of first contact approached I was distressed to find that the only shade which I could use with the double-image micrometer eye-piece was too dense to enable me to see well unless the clouds became thinner. In order to be ready I put the position-circle so that the wire (already put parallel to the line of separation of images) would cut off a small segment at the point of computed contact, and I placed the micrometer at the reading 16^r.204, which I had found to be that of the coincidence of images. Whenever I could see the Sun well through the shade the definition was excellent, and the clock kept the point of contact steadily in its proper position relatively to the wire. The thin clouds which were passing sometimes almost totally obscured the Sun, but I found that I could not attempt to observe without the shade, and with the shade I feared I might lose the first contact. The seconds were called every ten seconds of the chronometer by my wife, who acted as recorder, and I had the chronometer on a high stand so near me that I could keep the count without any interference by the beats of the equatorial driving-clock. My recollection is very distinct that the time 10^s.0 corresponds very nearly to actual first contact. At 12^s.0 I could see the indentation very plainly, and at 14^s.0 it was about what I had a few days previously noticed to be the indentation after 4 or 5 seconds in the case of an artificial transit in which relative sizes and motion were accurately represented. I am quite sure that the time recorded, 1^h 55^m 10^s.0, by BOND 290 is, although uncertain, not to exceed 5^s out, and I doubt whether it is really more than 1^s or 2^s out, although I felt at the time that it was uncertain, and made the first record of it as "very uncertain." The power was high, and when I could see the definition was excellent, and I tried to catch sharply the instant of contact. What I saw at 10^s.0, and supposed to be the planet, proved by following to be such, and I noticed the comparative indentation as above stated. I therefore finally marked out "very," and let the record stand as "uncertain." These explanations are necessary, because my practice with the artificial transit, less than a week ago, led me to expect to be able to catch the instant of first contact almost to the nearest second.

As soon as the first contact had been recorded, I proceeded to measure the cusps—which were excessively faint. I took the time from the chronometer right before me, and whenever I thought I had a contact of points, I noted the instant and called to the recorder the time to be recorded and then the readings of the micrometer. In order to be sure that the records were correctly made, I had her call them back to me as soon as they were written down.

I went on this way making measures of what I saw faintly and supposed to be the cusps, until all at once a break in the cloud showed me the cusps plainly, and that I had for the last preceding few measures perhaps been measuring probably the diameter of the planet, as the limb of the Sun had not in some of the measures been visible to the border, as the bright glimpse showed me. I succeeded in getting one more good measure, the last one recorded of the actual cusps. I think that the first four measures belong to the actual cusps. I am sure that the first two do, and I have as yet no means of knowing whether the next two do or not. Just when I lost the actual cusps and commenced measuring points further in, I know not. I did not have time to look at the record, and the break in the clouds at about 2^h 17^m first showed me that I was then attempting to measure almost, if not exactly, the diameter of Venus.

After the last measure recorded, I immediately placed the micrometer at 16^r.204 for coincidence of images, as I saw that second contact was near. The light was feeble, and at the instant recorded I saw an unbroken line of light clear across and exceedingly thin. I did not see any black ligament preceding its formation, the space between the cusps simply becoming brighter, and when I first noticed it, clear across and unbroken, I supposed it to be a momentary flash, and I did not fix upon the instant for a few seconds, not until I was sure the planet was fairly upon the Sun. The time was recorded as very uncertain, as I expected to see the final formation of this line

very sharply. I afterwards marked out "very uncertain" and let it stand as "uncertain," by which I wish to be understood that I think it is correct within 5 seconds. It may possibly be 10^s late, but I feel confident that 5^s is a liberal estimate of its uncertainty. At $2^h 21^m 35^s$ light appeared between the cusps, and during the period of 30^s the line of light across was broken and hazy, and hence on this account also I felt uncertain in regard to the instant of contact.

Although the band of light was faint, I think the measures of its width are pretty good. It was difficult for me to fix upon the exact position in the measures, and I have given the reading of the position circle when the measures of distance were taken. The readings of the circle are given first for all the measures before the revolutions of the screw. The second set of measures were stopped at last by dense clouds. The sky was completely overcast, and there was no hope from the immediate prospect of our being able to see anything more of the transit. About an hour before the last contact, the wind shifted to the northwest, and the clouds began to disperse so that it was again possible to observe the Sun, although through thin clouds and haze. The change of wind brought thin dust haze, and the definition was not good, although the brilliancy was sufficient to enable me to see well and to make measures. The measures of the width of the band of light between the limbs of the Sun and of Venus preceding the third contact are pretty good. I kept the count of time carefully from the chronometer directly in front of me, and at the instant when I succeeded in getting a proper contact, I called the time and read the micrometer. The times and readings given were recorded by my wife, and by her called back to me after she had written them down. I ceased measuring about five minutes before the third contact, so as to be fully prepared to observe this contact with great care, the brightness being now pretty good, what I suppose to be meant by the designation "average" referred to in the instructions to observers. I placed the micrometer at the coincidence of images, and the driving-clock held the limb steadily in the middle of the field. I took the count from the chronometer directly in front of me, and the recorder called every ten seconds. As the planet approached the limb of the sun very closely the narrow band of light appeared disturbed. At $6^h 11^m 6^s.0$ a narrow dark band shot across, connecting Venus with sky. But immediately, perhaps as soon as one second after its formation, it ceased, and the line of light between Venus and the edge of the Sun was until contact distinct, although at times broken by dark shadows fitting across it. At the exact instant recorded the line of light was suddenly broken, and Venus was connected with the sky. The cusps were quite sharp, and nothing more than a very slight apparent dark ligament was formed.

In fact, so far as I could judge, the amount of the blunting of the cusps was not more than could be attributed to the want of perfect definition in the telescope.

The time of the third contact as finally taken, and the phase as described, which I supposed to be the amount of actual contact, viz, $6^h 11^m 30^s.5$, I am confident is not in error half a second, since the apparent congelation of the fine line of light was almost instantaneous. The space between the cusps did not at once become black, but was illuminated by a sort of twilight for a period of 15 or 20 seconds.

I next proceeded to measure the distance between the cusps, and although there was considerable motion I was careful in making the contacts and keeping the time count (the tens being called by the recorder). At the time when the measures ceased, the cusps had become so blunt that accurate distances could not be measured, and I then placed the micrometer at the reading for coincidence of images so as to be ready to observe the last contact. The limb of the Sun was quite unsteady and undulating, but the indentation of the planet was easily seen and kept in view when very small. I kept my eye fixed upon it until the outline of the sun was perfect, and estimated the contact from the instant when it had ceased to be discernible. I consider the time to be accurate to a second. The recorder made note of the seconds and fractions called at the contacts, and I myself wrote down an independent note of the seconds and fractions. The minutes were immediately identified and recorded, except in the case of the first contact. When I came to identify the minute the chronometer had passed 56^m , and I wrote down 56, but immediately noticed that it had just passed $56^m 10^s$, and hence I changed the record to 55^m . I did not think that more than a minute had elapsed between the observation and the identification of the minute, but it is possible that such was the case and that the minute should have been 54. The seconds I am sure of, and it can be known hereafter whether the recorded minute is correct. I do not now recall any cir-

cumstance not mentioned which might be necessary to explain any of these observations. I have written out this more extended memorandum, this Wednesday evening, December 9, 1874, so that not yet knowing what may have been found by others, I can give an unbiased statement in regard to these observations.

JAMES C. WATSON, *Observer.*

P. S.—I forgot to mention that several times I noticed that the sun appeared brighter at the limb of Venus than at a little distance from it.

THURSDAY, *December 10.*

Clouds prevented observations for value of one revolution of micrometer screw of double-image micrometer.

SATURDAY, *December 12, 1874.*

On reflection, it occurs to me that I did not observe as contacts corresponding actual phases at second and third contacts on the 9th. At second contact I recorded (as the contact) the time when the line of white solar light was complete clear across and remained so. This line of light was very thin, but it certainly continued distinct and unbroken. The passing haze made the brilliancy irregular, but still there was sufficient light to enable me to see it without interruption. If the actual contact was when the line of light was occasionally broken in places and not steadily complete, this contact was earlier than that which I recorded; just how much I cannot say, because I recorded the instant when I considered it to be complete, and I was in a state of uncertainty about it for several seconds. The first gleam of light between the cusps was 30 seconds before the time recorded, and the whole period during which the space connecting the planet with the sky was filled with hazy light was, I think, fully this interval. At the third contact I was familiar with the phenomena to be expected, and I then recorded two instants sharply. As already stated in the note of the observation, I first saw, as the planet approached the limb of the Sun, the line of light momentarily broken $24^{\text{s}}.5$ before the formation of the distinct cusps and the joining of the planet and the sky. During this interval there were flitting shadows along the line of light. I suppose the actual contact to be that when I saw the cusps suddenly formed. At this contact the brilliancy was good, but the definition was not so sharp as in the forenoon.

The first interruption of the line of light was by a single very narrow band, and after its disappearance the shadows which flitted back and forth were radial to Venus. After the sudden formation of the cusps there was between them a very distinct gleam of uniform grayish light, which I called at the time a sort of twilight. I did not see any such light at second contact, but this may be owing to the interruption of the light by the clouds and the shade glass which I was obliged to use. The whole transformation was so gradual that to fix sharply distinct phases required a brighter image than I could obtain at this (second) contact, but what I did see convinces me that the phenomena were repeated in reverse order. And I ought to record here, while the recollection is fresh, that although the observer by practice upon the artificial transit may know pretty well what to expect, there is a lack of definiteness that makes him feel that the moment fixed upon is quite uncertain, and the time during the observation in which he feels this uncertainty seems to be much longer than it really is. Hence, I record here my conviction that practiced observers will assign probable errors to their observed times within which the actual time is sure to be. I think further that the last two contacts will be observed much more sharply than the first two, even when the seeing is equally good in both cases. In my own case, I have not a doubt as to the sharpness of my determinations of the times of the third and fourth contacts, and I think that the forenoon observations, although under more disadvantageous circumstances, are to be relied upon within the limits of error assigned. The first contact had certainly taken place at the instant recorded, and the time given, if in error, is too late. At the second contact the line of white light was certainly completely established at the time recorded, and it might have been so completed a few seconds earlier, perhaps 5 seconds earlier.

The difference between the phenomena of the actual transit and the artificial transit, I feel confident, are to be explained by the action of the atmosphere of Venus, and I think it cannot be determined which of the different phases observed is the actual contact of limbs until the effect of this atmosphere is carefully investigated by means of the observations of the actual transit. The effect of the chromosphere and solar corona must also be considered in this connection.

Notes of Prof. C. A. Young and Mr. T. P. Woodward.

DECEMBER 9.

First contact observed with DILLON; sure at $2^h 9^m 30^s$; true time, 10^s earlier (?) Instrument small CLARK telescope; power, 25, lowest belonging to the instrument; seeing bad; limb undulating; lightest shade glass.

Second contact, $2^h 37^m 3^s \pm 05$; magnifying power next higher than preceding = 80; seeing steady, but very faint through haze; no black drop seen; planet fairly clear of Sun's limb at recorded time.

The comet seeker not having a sufficiently light shade glass, impossible to observe the contacts with it.—T. P. W., (not) observer.

h. m. s.	
6 26 38 \pm 5	C. A. Y.—Small CLARK.
6 26 56	T. P. W.—Comet seeker.

C. A. Y.: Recorded the first formation of any black band between Venus and the limb of the Sun. Perhaps somewhat early; limb of Sun very tremulous; power same as at second contact (80); no distinct connection between Venus and Sun's limbs for 15^s more; appearance not like the artificial transit on account of the extreme tremulousness of limb.

h. m. s.	
6 53 54 \pm 2	C. A. Y.—CLARK glass.
6 53 45	T. P. W.—Comet seeker.

C. A. Y.: Limb tremulous, but egress unexpectedly distinct; magnifying power same as at second and third contacts, *i. e.*, 80.

T. P. W.: Third contact.—Observed Venus apparently touching the Sun at $6^h 26^m 30^s$, and after verifying the time from the chronometer looked again through the comet seeker, when I saw she was still separated by a slight line. After several apparent connections and separations I took the time at $6^h 26^m 56^s$, when she was certainly in contact and perhaps had been for 5 seconds.

Both disks were well defined and steady. The secondary spectrum and the size of the image are all that was disadvantageous. No drop was seen; magnifying power, 40.

Fourth contact.—Followed Venus as long as possible, and recorded the time when she disappeared. I can assign no probable error; seeing was good and steady; secondary spectrum not so apparent; observation good; power, 40.

Fuller remarks on observations of contacts recorded above.

C. A. Y.: First contact.—At the time noted, $2^h 9^m 30^s$, I perceived, or thought I perceived, a modification of the Sun's limb at the precise point I was watching. I continued my count, however. By $2^h 9^m 40^s$ the mark was unmistakable, and by $2^h 9^m 45^s$ was plainly a *notch*, which increased in depth until $2^h 10^m 0^s$, when I ceased counting and recorded my observation as above. The Sun was seen through clouds, and there was a good deal of disturbance of the limb, but with my very light shade glass the image was quite bright. I now hardly think the contact was visible in my telescope 10^s before the recorded time. The word "probably" was erased in my notes and the "?" added at the time of the second contact, my conclusion being modified by the rapidity of the motion then observed.

Second contact.—The time recorded, $2^h 37^m 3^s$, is that at which light first broke through between the planet and the Sun's limb. There were one or two suspected glimmerings about two seconds earlier. The separation did not become *permanent, i. e.*, unbroken by occasional momentary dark fringes, until 10^s or 15^s later. The clouds were somewhat thicker than at the time of first contact, but the image of the Sun was very steady, and the seeing and definition good. The image was white, somewhat fogged by the clouds, but bright enough for accurate observation.

Third contact.—The time recorded undoubtedly belongs to a phase of the phenomenon earlier than that described in the "instructions," which, unfortunately, circumstances prevented me from looking at until after the observation.

The time recorded, $6^h 26^m 38^s$, is that at which several patches or lines of *darkness* first appeared to connect the limbs of Venus and the Sun. A single such connection was formed for an instant

at 6^h 26^m 30^s, but instantly disappeared. Light did not cease to glimmer through the interval until 6^h 26^m 53^s, at which time the last streak of light I saw flashed across; so that after that time the dark band between the limbs of Venus and the Sun became distinct and permanent. I saw no "sudden congelation" and no black drop.

There were no clouds at the time, but there was a thick haze of yellow dust, and the air was very unquiet, so that the seeing was decidedly bad. I think, however, my $\pm 5^s$ is an exceedingly liberal estimate of the probable error of the phase observed, and that 6^h 26^m 53^s corresponds pretty closely to the phase contemplated in the instructions—within 5 seconds certainly.

Fourth contact.—I am *sure* I saw the planet at 6^h 53^m 50^s, and that it was not visible at 6^h 53^m 58^s. I am pretty confident I still perceived it at 6^h 53^m 53^s, and that I could not perceive it at 6^h 53^m 56^s. Hence I recorded the time as stated in the notes, 6^h 53^m 54^s. The light was abundant, the air tremulous, and the seeing bad, causing the image of the Sun to look like a circular saw; but, as noted, the egress was unexpectedly definite, for until the planet left the Sun the serrations at the point of contact were plainly blunted.—C. A. Y.

The magnifying powers were determined December 11 by measuring the diameter of the image of the object glass formed at the eye-piece. Lowest power of CLARK telescope, 25; second power, 80; comet seeker, 40.—C. A. Y.

December 10.—Chronometer comparisons. Chronograph started at 17^h 25^m 0^s. Marked 17^h 27^m 0^s.

	h. m.				
NEGUS	3	54	00	10	20 30.
BOND	4	35	30	40	50 60.
DILLON	4	52	00	10	20 30.

MOLLOY POINT, KERGUELEN ISLAND.

Commander Ryan's record of his contact observations.

Chronometer 827 MURRAY L. M. T. First contact 6^h 41^m 35^s.5 excellent.

* * * * *
Took off colored eyepiece.

7^h 13^m 41^s, time when first seen after second contact, cloud having passed. Bright strip too wide to be called contact.

CAMPBELLTOWN, TASMANIA.

Captain Raymond's record of his observation of third contact.

Third contact.—Time, 3^h 40^m 50^s; dim and cloudy; planet pear-shaped at the time of contact; limb of Sun unsteady; not a satisfactory observation on account of clouds. I took the time when the thin, shadow-like link connecting limbs of planet and Sun seemed to cougeal, as near as possible.—C. W. R.

[A pencil sketch of the third contact was given by the observer, but it is not possible to reproduce it in such a way as to assist in judging the observation. Contact seems in it to be complete.]—EDITOR.

QUEENSTOWN, NEW ZEALAND.

Professor Peters' notes.

December 9, 1874.—Adjusted focus and position-micrometer upon solar spot. The images seem to coincide at 14^s.02; position-micrometer, 245^o.

Turned the position-micrometer upon 196^o, which should make the wire tangent to the Sun's limb where first contact occurs.

BOND 335 chronometer was used [contact I should come at 18^h 13^m 16^s chronometer]; 18^h 0^m, clouds are racing with blue patches between them; 18^h 8^m, Sun near edge of a big cloud, but still behind it; 18^h 14^m 0^s, first indentation perceptible, but very uncertain.

18^h 43^m 48^s, first internal contact, well observed; no indications whatever of irradiation or other physical phenomena.

Additional remarks for observation of first internal contact: The slit of the double-image micrometer was placed, a little before, at right angles to the Sun's limb, the two images apart. The bright rim of Venus that had been seen outside the sun for several minutes before already, increased in brightness. But this increase in brightness was at once more sudden, and almost instantaneous, that is to say within a second or two. This moment was taken as the true contact. Soon after, the Sun's light was distinctly seen entirely surrounding the disk of Venus, or Venus was seen entirely upon the Sun. [Note written at 20^h 0^m chronometer.]

All the observations are made with lowest of the two powers of double-image micrometer [= 90 diameters].

21^h 5^m.—After little rain showers and clouds, cleared up again; wind pretty strong from SW. Sun's and Venus' limbs very much undulating. Try to measure distance of Venus' and Sun's limbs.

22^h 12^m 15^s.—A moment through clouds; Venus already on the limb of Sun; third contact passed.

22^h 43^m 30^s.—The Sun comes out suddenly from behind a dense cloud; the limb is strongly undulating, so that last external contact, even if it had not happened one minute before, could not have been remarked.

The time reductions of the preceding observations of contact are shown in the following tables. But the times given are not at all to be regarded as the definitive moments of contact to be deduced from the remarks of the observers, but only as times so near those of contacts that the necessary reductions can be applied differentially without serious trouble. Where only one time is noted by the observer that time is quoted in the column chronometer time. Where several are noted by the observer one is selected for reduction.

The determination of the times which should be finally concluded as those of contact necessarily form a separate branch of the work. All that is here attempted is to present the material in such a shape that the necessary discussions can be undertaken with the greatest facility.

The Greenwich times still require a symbolic correction for the provisional longitude:

WLADIWOSTOK.

Contact.	Observer.	Aperture of instrument.	Power.	Chronometer time.			Correction of chronometer.			Wladiwostok mean time.			Wladiwostok west of Greenwich.			Greenwich mean time.			Greenwich sidereal time.				
		Inches.		h	m	s	h	m	s	h	m	s	h	m	s	h	m	s	h	m	s		
I.	HALL.....	5	140	1	44	35.0	—	3	11	45.9	22	32	49.1	—	8	47	30.9	13	45	18.2	6	55	51.4
II.	HALL.....	5	140	2	10	52.0	—	3	11	45.9	23	0	6.1	—	8	47	30.9	14	12	35.2	7	23	12.9
I.	WHEELER..	3	30	10	30	47.0	+	2	7.4	22	32	54.4	—	8	47	30.9	13	45	23.5	6	55	56.7	
II.	WHEELER..	3	30	10	58	5.0	+	2	7.4	23	0	12.4	—	8	47	30.9	14	12	41.5	7	23	19.2	
III.	WHEELER..	3	30	2	49	50.5	+	2	7.8	2	51	58.3	—	8	47	30.9	18	4	27.4	11	15	43.1	

REMARK.—The difference of 0^s.2 between these times and those given in the body of the work arises from the application of a different chronometer correction.

It is assumed that Hall's chronometer time of second contact requires to be increased by one minute, as suggested in his notes.

APPENDIX.

NAGASAKI.

Contact.	Observer.	Aperture of instrument.	Magnifying power.	Recorded time.	Correction of chronometer.	Nagasaki mean time.	Nagasaki west of Greenwich.	Greenwich mean time.	Greenwich sidereal time.
				h m s	s	h m s	h m s	h m s	h m s
I.	DAVIDSON	5	...	15 36 52	— 11.5	22 26 7.1	15 20 29.4	13 46 36.5	6 57 9.9
II.	DAVIDSON	5	...	16 4 46.5	— 11.5	22 53 57.0	15 20 29.4	14 14 26.4	7 25 4.4
III.	DAVIDSON	5	...	19 53 52	— 11.5	2 42 25.0	15 20 29.4	18 2 54.4	11 14 9.9
I.	TITTMANN....	3	60	10 26 17	+ 4.7	22 26 21.7	15 20 29.4	13 46 51.1	6 57 24.5
II.	TITTMANN....	3	60	10 52 20	+ 4.7	22 52 24.7	15 20 29.4	14 12 54.1	7 23 31.8
III.	TITTMANN....	3	60	2 42 40	+ 4.8	2 42 44.8	15 20 29.4	18 3 14.2	11 14 29.7

NOTE.—Careful inquiry has failed to throw any light upon the cause of the discordance of a minute and a half between the times of second contact recorded by Professor DAVIDSON and Mr. TITTMANN. There is little doubt that an error was made by one or both observers in recording the chronometer time.

PEKING.

Contact.	Observer.	Aperture of instrument.	Magnifying power.	Recorded time.	Correction of chronometer.	Peking mean time.	Peking west of Greenwich.	Greenwich mean time.	Greenwich sidereal time.
		Inch's.		h m s	h m s	h m s	h m s	h m s	h m s
I.	WATSON	5	165:	1 55:10.0	+ 7 3 ⁸ 33.0	21 32 43.0	16 14 12.1	13 46 55.1	6 57 28.5
II.	WATSON	5	165:	2 22 5.0	+ 7 3 ⁸ 33.2	22 0 38.2	16 14 12.1	14 14 50.3	7 25 28.3
III.	WATSON	5	165:	6 11 30.5	+ 7 3 ⁸ 33.9	1 50 4.4	16 14 12.1	18 4 16.5	11 15 32.2
IV.	WATSON	5	165:	6 38 44.0	+ 7 3 ⁸ 34.0	2 17 18.0	16 14 12.1	18 31 30.1	11 42 50.3
I.	YOUNG.....	2.5	25	2 9 30	+ 7 23 8.0	21 32 38.0	16 14 12.1	13 46 50.1	6 57 23.5
II.	YOUNG.....	2.5	80	2 37 3 ± 5	+ 7 23 8.0	22 0 11.0	16 14 12.1	14 14 23.1	7 25 1.0
III.	YOUNG.....	2.5	80	6 26 38 ± 5	+ 7 23 7.5	1 49 45.5	16 14 12.1	18 3 57.6	11 15 13.3
IV.	YOUNG.....	2.5	80	6 53 54 ± 2	+ 7 23 7.5	2 17 1.5	16 14 12.1	18 31 13.6	11 42 33.7
III.	WOODWARD .	3	40	6 26 56	+ 7 23 7.5	1 50 3.5	16 14 12.1	18 41 5.6	11 15 31.3
IV.	WOODWARD .	3	40	6 53 45	+ 7 23 7.5	2 16 52.5	16 14 12.1	18 31 4.6	11 42 24.7

Note.—From Professor Watson's notes, p. 151, it is supposed that his minute of first contact should have been 54.

MOLLOY POINT, KERGUÉLEN.

Contact.	Observer.	Aperture of instrument.	Magnifying power.	Recorded time, MURRAY 827 mean time.	Correction of MURRAY 827 mean time.	Kerguelen mean time.	Kerguelen west of Greenwich.	Greenwich mean time.	Greenwich sidereal time.
		Inches.		h m s	m s	h m s	h m s	h m s	h m s
I.	RYAN	5	..	18 41 35.5	— 2 16.2	18 39 19.3	19 19 40.4	13 58 59.7	7 9 35.1
II.	RYAN	5	..	19 13 41	— 2 16.1	19 11 24.9	19 19 40.4	14 31 5.3	7 41 46.0

CAMPBELLTOWN.

Contact.	Observer.	Aperture of instrument.	Magnifying power.	Recorded time, PORTER 118 mean time.	Correction of PORTER 118 mean time.	Campbelltown mean time.	Campbelltown west of Greenwich.	Greenwich mean time.	Greenwich sidereal time.
III.	RAYMOND.	Inches. 5	90	h m s 3 40 50	m s - 1 41.1	h m s 3 39 8.9	h m s 14 9 59.9	h m s 17 49 8.8	h m s 11 0 22.0

QUEENSTOWN.

Contact.	Observer.	Aperture of instrument.	Magnifying power.	Recorded time, BOND 335 sidereal time.	Correction of BOND 335 sidereal time.	Queenstown mean time.	Queenstown west of Greenwich.	Greenwich mean time.	Greenwich sidereal time.
I.	PETERS..	Inches. 5	90	h m s 18 14 0	s + 43.9	h m s 1 4 10.0	h m s 12 45 19.6	h m s 13 49 29.6	h m s 7 0 3.5
II.	PETERS..	5	90	18 43 48	+ 43.9	1 33 53.2	12 45 19.6	14 19 12.8	7 29 51.5
III.	PETERS..	5	90	22 12 15	+ 43.8	5 1 45.9	12 45 19.6	17 47 5.5	10 58 18.4
IV.	PETERS..	5	90	22 43 30	+ 43.8	5 32 55.8	12 45 19.6	18 18 15.4	11 29 33.4

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